

Problems in Mathematics & Experiments with Mathematica

2. Elementary functions

2.5 Exponential and logarithmic functions

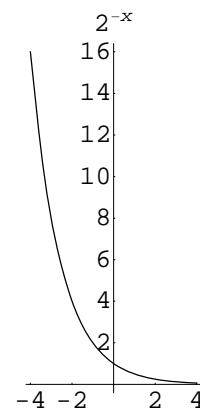
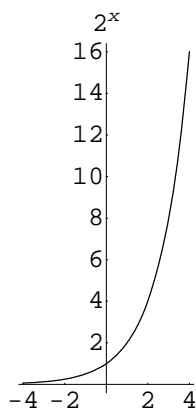
Exponential function

- **Definition**

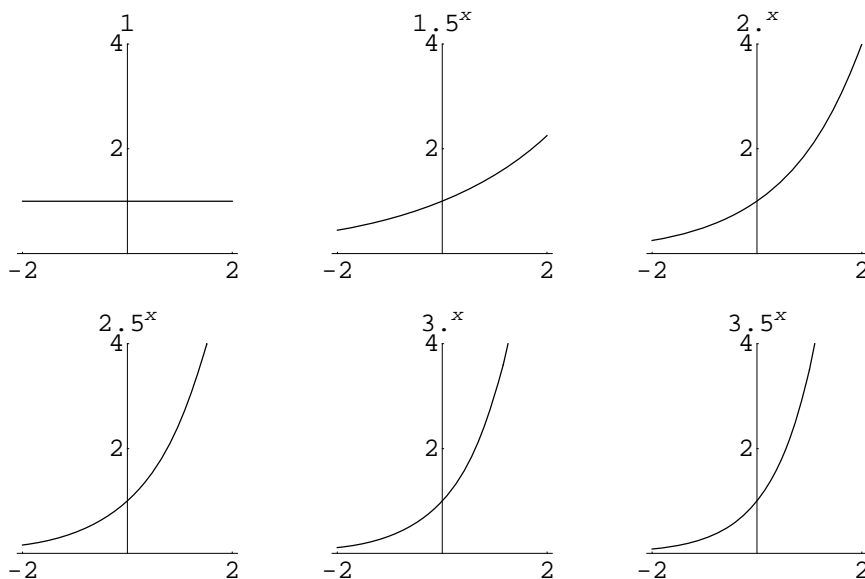
If, in the power a^x , the basis $a > 0$ is fixed and the exponent $x \in \mathbb{R}$ is a variable, then we have an exponential function. According to the properties of the powers, it has the following properties:

- $a^x > 0$ for every $x \in \mathbb{R}$;
- a^x is increasing if $a > 1$;
- a^x is decreasing if $0 < a < 1$.

- **Graphs**

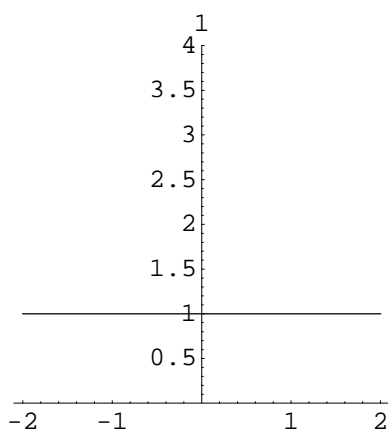


- The growth of exponential functions

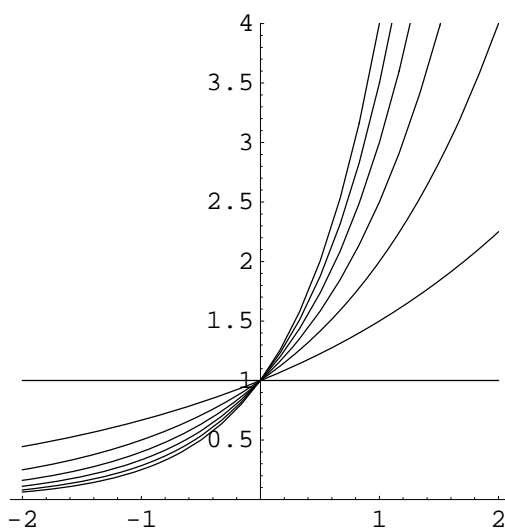


- Animation (electronic version only)

To animate, double click on the first frame.



- The graphs together



- Statements of the plots (electronic version only)

Logarithmic functions

• Definition

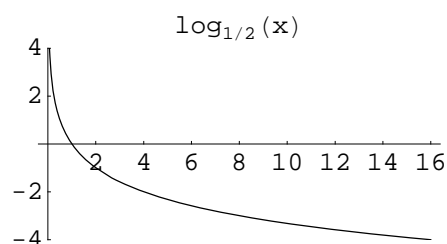
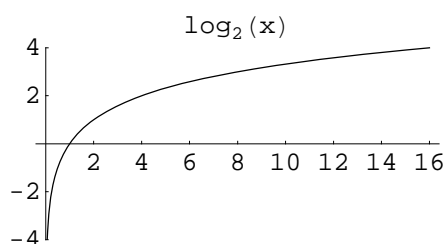
The function a^x is strictly monotone for $0 < a < 1$ or $1 < a$. Its inverse exists and is denoted by $\log_a(x)$.

By definition,

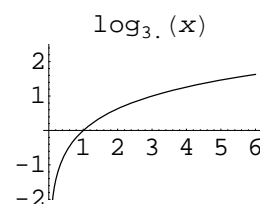
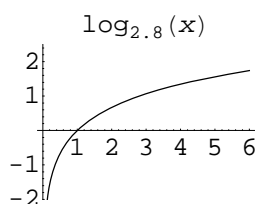
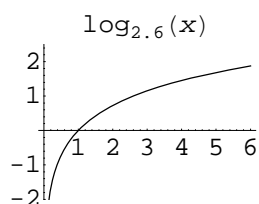
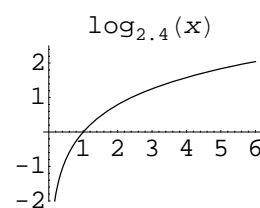
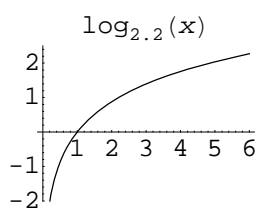
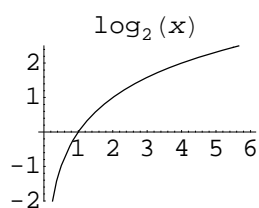
$$\log_a(a^x) = x \text{ for every } x \in \mathbb{R};$$

$$a^{\log_a(x)} = x \text{ for every } x > 0;$$

• Graphs

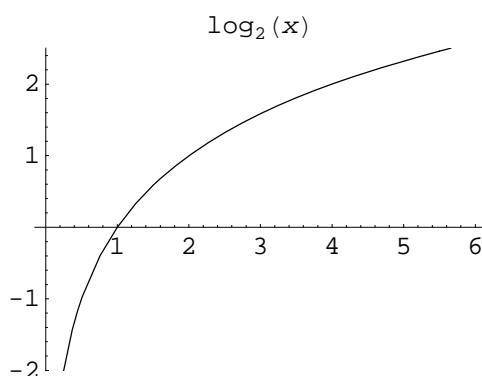


• The growth of logarithmic functions

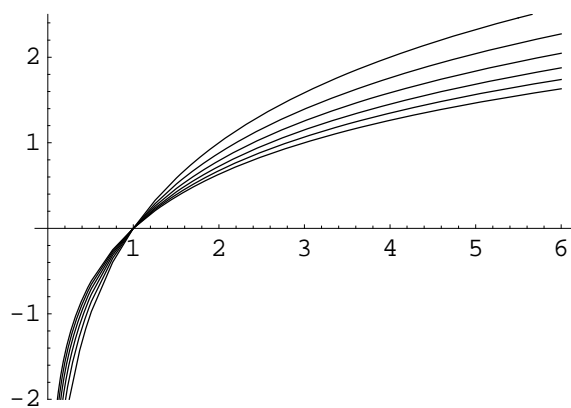


• Animation (electronic version only)

To animate, double click on the first frame.



- The graphs together



- Statements of the plots (electronic version only)

Mathematica statements

InputForm	StandardForm	TraditionalForm
<code>a^x</code>	a^x	a^x
<code>Log[2, x]</code>	<code>Log[2, x]</code>	$\log_2(x)$