

Problems in Mathematics & Experiments with Mathematica

2. Elementary functions

2.8 Elementary transformations

Theory

The elementary transformations are summarized below:

Formula	Transformation of the graph
$f(x - p) + q$	shift the graph by the vector (p, q)
$a f(x), \quad a > 0$	$a > 1$: proportionally enlarges the graph in y – direction; $0 < a < 1$: compresses the graph
$-f(x)$	mirroring to the x – axis
$f(cx)$	$c > 1$: proportionally compresses the graph in x – direction; $0 < c < 1$: enlarges the graph
$f(-x)$	mirroring to the y – axis
$-f(-x)$	mirroring to the origin
$af(cx - p) + q$	general composed transformation

Prove the above properties. For graphical illustrations, see the solved problems.

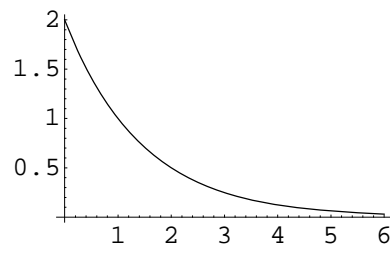
Simple Exercises and problems

PROBLEM 2.8.1 Transformations on graphs

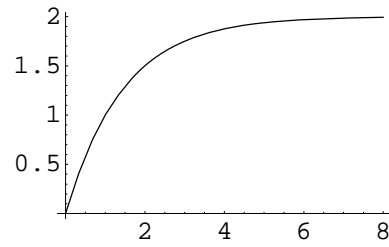
Plot the following functions if $f(x)$ is one of those plotted below:

$$\begin{array}{lll}
 g(x) = 2f(x - 1) - 2; & g(x) = -f(x + 1) - 2; & g(x) = f(x - 2) + 1; \\
 g(x) = f(2x); & g(x) = -f(x/3) - 2; & g(x) = f(x - 2) + 1;
 \end{array}$$

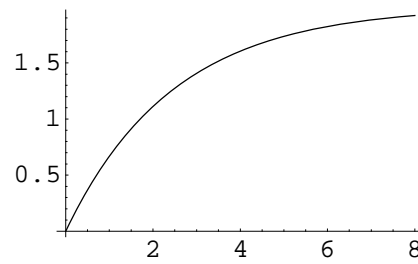
(1)



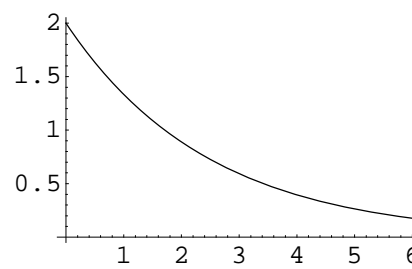
(2)



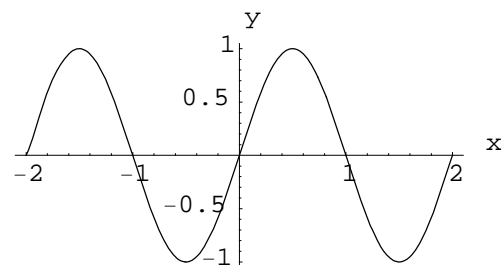
(3)



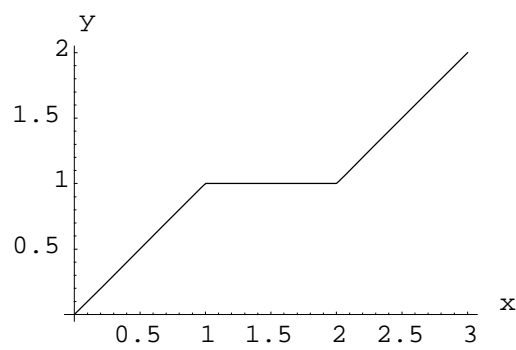
(4)



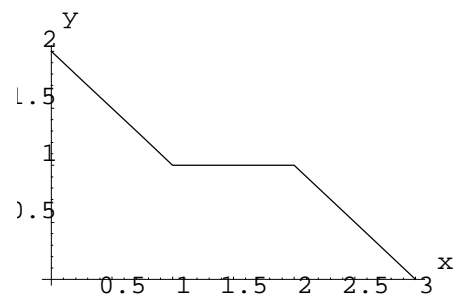
(5)



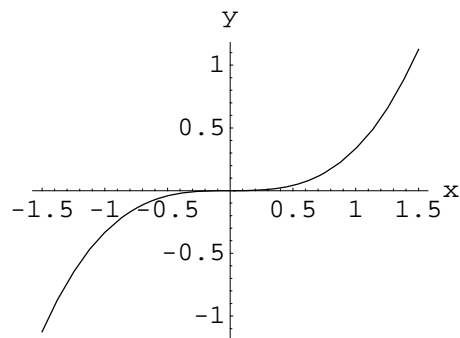
(6)



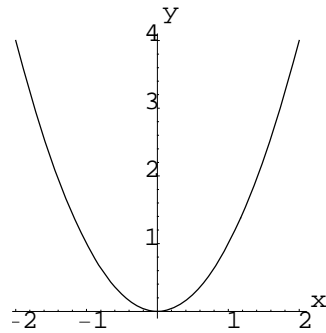
(7)



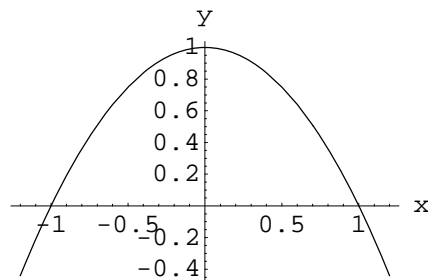
(8)



(9)



(10)



■ Mathematica initialization

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<< "Graphics`Graphics`"
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SOLVED PROBLEM 2.8.2 Translation by the vector (p,q)

Plot the graph of the function.

$$f(x) := \sin(x - 1) + 2;$$

○ SOLUTION

The graph of the function

$$g[x] := \sin[x]$$

is translated by the vector

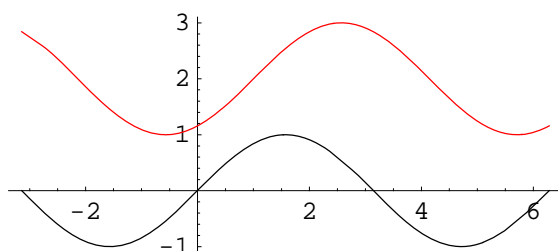
$$\{p, q\} = \{1, 2\};$$

Plot the original and the transformed functions:

```

x0 = -π; x1 = 2 π;
Plot[{g[x], q + g[x - p]}, {x, x0, x1}, AspectRatio → Automatic,
PlotStyle → {{Hue[1, 1, 0]}, {Hue[1]}}];

```



• Animation (electronic version only)

○

PROBLEM 2.8.3

Plot the graph of the following functions.

(1) $f(x) := x^2 + 2x$; $g(x) := x^2 + 2x + 2$; $h(x) := 4x - x^2$;

(2) $f(x) := \cos(x)$; $g(x) := \sqrt{x-1}$; $h(x) := \log(x-2)$;

SOLVED PROBLEM 2.8.4 Transformation $a f(x)$

Plot the graph of the function

$$f(x) := 2x^3$$

○ SOLUTION

To obtain the graph of $f(x)$, each y-coordinate of the graph of the function

$$g[x] := x^3$$

has to be multiplied by

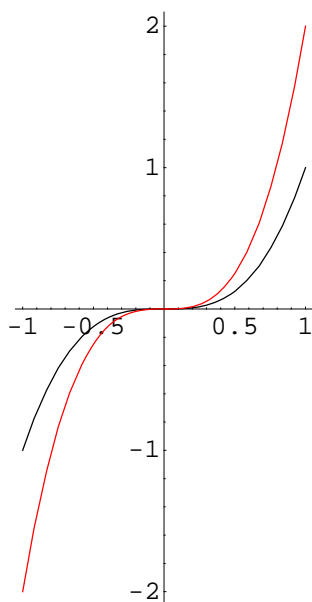
$$a = 2;$$

Plot the original and the transformed functions:

```

x0 = -1; x1 = 1;
Plot[{g[x], a g[x]}, {x, x0, x1},
  AspectRatio → Automatic, PlotRange → All,
  PlotStyle → {{Hue[1, 1, 0]}, {Hue[1]}}];

```



• Animation (electronic version only)

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PROBLEM 2.8.5

Plot the graph of the following functions:

(1) $f(x) := 3x^2$; $g(x) := \log_{\frac{1}{2}}(x)$; $h(x) := 2^{(x+1)}$;

(2) $f(x) := -x^2$; $g(x) := -3^x$; $h(x) := -2\sin(x)$;

SOLVED PROBLEM 2.8.6 Transformation $f(cx)$

Plot the graph of the function:

$$f(x) := \sin(3x);$$

○ SOLUTION

To obtain the graph of $f(x)$, x-coordinates of the graph of the function

$$g[x] := \sin[x];$$

is divided by

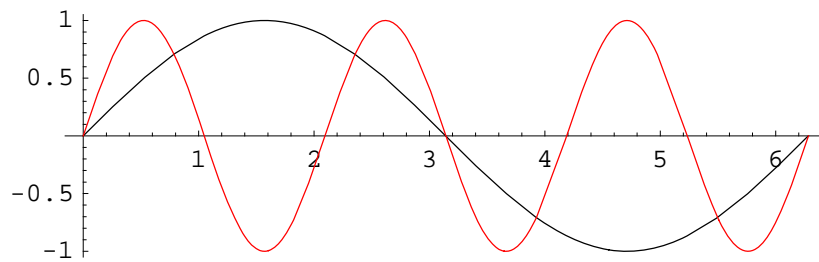
$$c = 3;$$

Plot the original and the transformed functions:

```

x0 = 0; x1 = 2 π;
Plot[{g[x], g[c x]}, {x, x0, x1},
  AspectRatio → Automatic, PlotRange → All,
  PlotStyle → {{Hue[1, 1, 0]}, {Hue[1]}}];

```



• Animation (electronic version only)

○

PROBLEM 2.8.7

Plot the graph of the following functions. Find the periods.

(1) $f(x) := \tan\left(\frac{x}{2}\right)$; $g(x) := \tan(3x)$; $h(x) := \cot(\pi x)$;

(2) $f(x) := \cos(\pi x)$; $g(x) := \cos(-x)$; $h(x) := \cos\left(\frac{x}{2}\right)$;

(3) $f(x) := \sin(-2x)$; $g(x) := \sin\left(\frac{x}{3}\right)$; $h(x) := \sin(3x)$;

Combinations of transformations

SOLVED PROBLEM 2.8.8 A transformation $a(f[cx - p] + q)$

Plot the graph of the following function:

$$f(x) := \frac{1}{2}(x-1)^2 - 2;$$

○ SOLUTION

To obtain the graph of $f(x)$, first, we use a transformation $a g(x)$ on

$$g[x] := x^2$$

with the constant

$$a = \frac{1}{2};$$

and then, shift the obtained graph by the vector

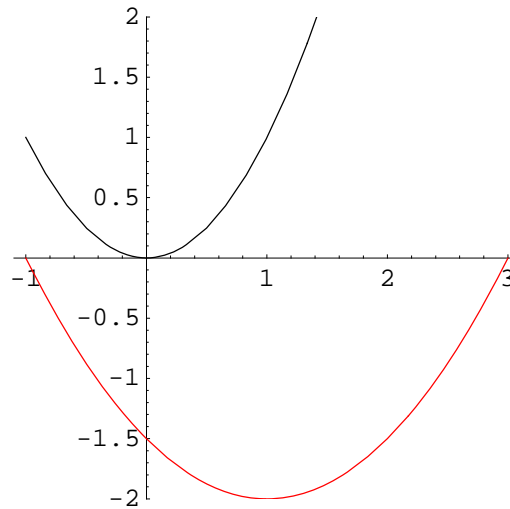
$$\{p, q\} = \{1, -2\};$$

Plot the original and the transformed functions:

```

x0 = -1; x1 = 3;
y0 = -2; y1 = 2;
Plot[{g[x], a g[x - p] + q}, {x, x0, x1},
  AspectRatio → Automatic, PlotRange → {y0, y1},
  PlotStyle → {{Hue[1, 1, 0]}, {Hue[1]}}];

```



• Animation (electronic version only)

○

PROBLEM 2.8.9

Plot the graph of the following functions.

(1) $f(x) := 3(x - 2)^3 - 1$; $g(x) := \sqrt{x - 1} + 3$; $h(x) := \log_{\frac{1}{2}}(x - 1) - 2$;

(2) $f(x) := \log_3(-x)$; $g(x) := \sqrt{-x}$; $h(x) := 2^{3x}$;

(3) $f(x) := 1 - 2^{2x}$; $g(x) := 3 - 2^{-x}$; $h(x) := 3^{-x} + 1$;

(4) $f(x) := 2(1 - 4^{-x})$; $g(x) := -3(2^x - 2)$; $h(x) := -2\left(\left(\frac{1}{2}\right)^x - 1\right)$;

(5) $f(x) := \frac{x - 1}{x + 2}$; $g(x) := \frac{x + 3}{1 - x}$; $h(x) := \frac{x + 1}{1 - 2x}$;
