3. Elementary Transformations

PROBLEM 1 LINEAR TRANSFORMATIONS OF THE FUNCTION VALUE

Sketch the graph of the given function via transforming the appropriate elementary functions!

- In these, along with the dependent variable, the range (of values) of the function is also transformed, while the domain stays the same.
- $y = f(x) \to y = f(x) + d$ transformation: vertical shift (d>0: upwards, d<0: downwards):

  a) $y = x^2+2$ b) $y = x^3-3$ c) $y = 0{,}5+\sin x$

  d) $y = \tan x - 1$ e) $y = 2^x+0{,}25$ f) $y = \sqrt{x} - 2$

- $y = f(x) \to y = c\,f(x)$, c>0 transformation: vertical stretch (c>1) or shrink (c<1):

  a) $y = 0{,}25x^2$ b) $y = 3x^3$ c) $y = 0{,}5\sin x$

  d) $y = 2\cot x$ e) $y = 2\cdot 0{,}5^x$ f) $y = 2\sqrt{x}$

- $y = f(x) \to y = -f(x)$ transformation: reflection to the x-axis:

  a) $y = -x^2$ b) $y = -x^3$ c) $y = -\cos x$

  d) $y = -\tan x$ e) $y = -2^x$ f) $y = -\sqrt[3]{x}$

PROBLEM 2 LINEAR TRANSFORMATIONS OF THE INDEPENDENT VARIABLE

In these, along with the independent variable, the domain (of definition) of the function is also transformed, while the range stays the same.

- $y = f(x) \to y = f(x+b)$ transformation: horizontal shift (b>0: to the left, b<0: to the right):

  a) $y = (x-1{,}5)^2$ b) $y = (x+3)^3$ c) $y = \cos(x - \frac{\pi}{4})$ d) $y = 2^{x-1}$ e) $y = \sqrt{x-2}$

- $y = f(x) \to y = f(ax)$, a>0 transformation: horizontal stretch (a<1) or shrink (a>1):

  a) $y = (0{,}5x)^2$ b) $y = \cos(\frac{1}{2}x)$ c) $y = \sin 2x$ d) $y = \tan\frac{x}{2}$ e) $y = \sqrt{3x}$

- $y = f(x) \to y = f(-x)$ transformation: reflection to the y-axis:

  a) $y = (-x)^2$ b) $y = \sin(-x)$ c) $y = e^{-x}$ d) $y = \log_2(-x)$ e) $y = \sqrt{-x}$

PROBLEM 3 COMBINED LINEAR TRANSFORMATIONS

- Sketch the graph of the given function via elementary transformation steps, starting from the appropriate elementary function:

a) $y = -2(x+1)^2+4$ b) $y = 0{,}5(x-1)^3-2$ c) $y = 1-2^{x+1}$

d) $y = 4+4^{-x/2}$ e) $y = 2\log_2(-x)-3$ f) $y = \log_{1/2}(2x)+1$

g) $y = 2-\sin(\frac{1}{2}x)$ h) $y = 2\cos 2x+1$ i) $y = 2\tan x-1$

j) $y = 2\sqrt[3]{x-1}-1$

- Modify the form of the *linear fractional function* and sketch its graph via elementary transformation steps, starting from the base function $1/x$:

a) $y = \dfrac{2x+1}{x}$ b) $y = \dfrac{x-1}{2x}$ c) $y = \dfrac{2x}{x+1}$ d) $y = \dfrac{2x-1}{2x+1}$ e) $y = \dfrac{7-3x}{x-2}$

- Plot the following functions:

$f(x\_) := 3(x-2)^3-1;\ g(x\_) := \sqrt{x-1}+3;\ h(x\_) := \log_{\frac{1}{2}}(x-1)-2;$

$f(x\_) := \log_3(-x);\quad g(x\_) := \sqrt{-x};\quad h(x\_) := 2^{3x};$

$f(x\_) := 1-2^{2x};\quad g(x\_) := 3-2^{-x};\quad h(x\_) := 3^{-x}+1;$

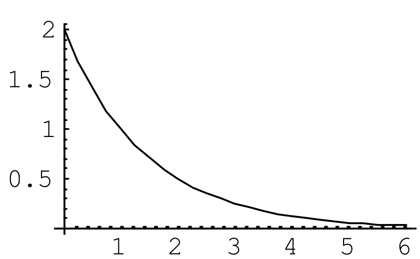
$f(x\_) := 2(1-4^{-x});\ g(x\_) := -3(2^x-2);\ h(x\_) := -2\left(\left(\frac{1}{2}\right)^x-1\right);$

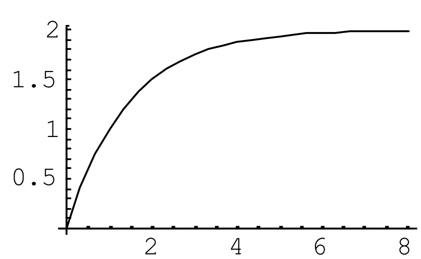
$f(x\_) := \dfrac{x-1}{x+2};\quad g(x\_) := \dfrac{x+3}{1-x};\quad h(x\_) := \dfrac{x+1}{1-2x};$

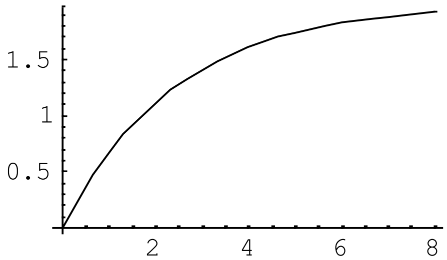
- Plot the functions *g(x)*, if *f(x)* is given by the following graphs:

$g(x)=2f(x-1)-2$; $g(x)=-f(x+1)-2$; $g(x)=f(x-2)+1$;

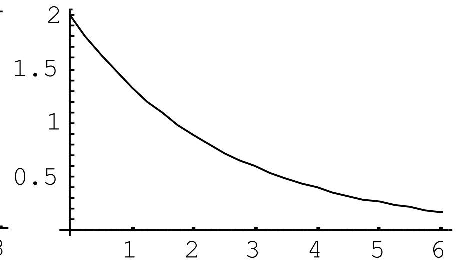
$g(x)=f(2x)$; $g(x)=-f(x/3)-2$; $g(x)=f(x-2)+1$;



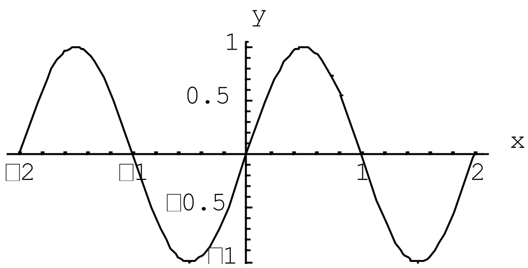




1.5
1
0.5
2
4
6
8



2
1.5
1
0.5
1
2
3
4
5
6



y
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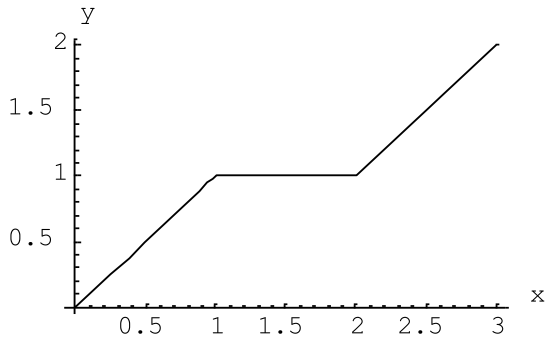
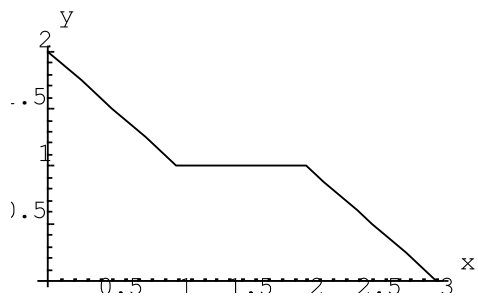
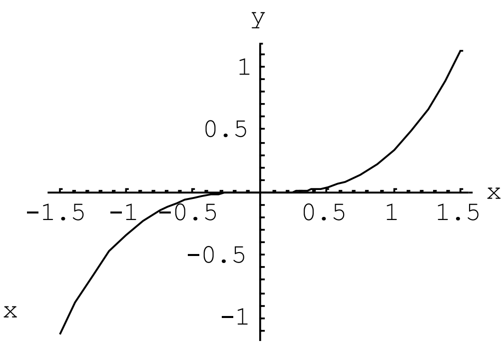
0.5
x
□2
□1
□0.5
□1
1
2



y
2
1.5
1
0.5
x
0.5
1
1.5
2
2.5
3



y
2
x



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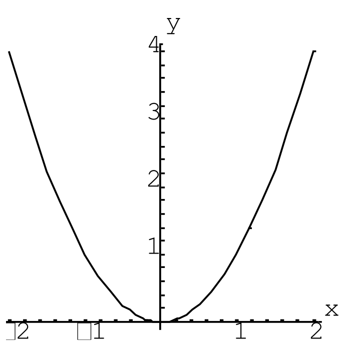
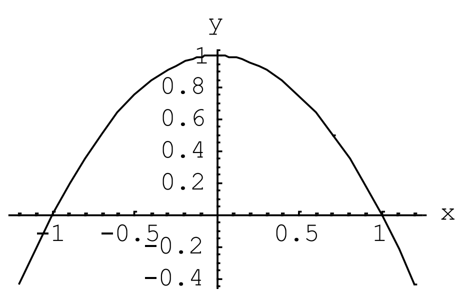
0.5
x
-1.5
-1
-0.5
0.5
1
1.5
-0.5
-1



y
4
3
2
1
x
1
2



y
1
0.8
0.6
0.4
0.2
x
-1
-0.5
0.5
1
-0.2
-0.4