M7. Ábrák programkódjai

2. Piaci modellek

2.1. Oligopóliumok

1. ábra

```
Row[{Manipulate[
   Plot[Max[0, b - d q], {q, 0, b / d + 0.5}, AxesLabel → {q, "P(q)"},
    PlotRange → {0, b + 0.5}, PlotStyle → Purple,
    ImageSize → {120, 120}], Column[{"Lineáris árfüggvény: ",
     ToString[TraditionalForm[P[q] == b - d q]]},
    Alignment → {Center, BaseLine}, ItemSize → {17, 1}],
   Delimiter, Control[{{b, 2}, 0, 10, ImageSize → Tiny,
     Appearance → {"Labeled"}}], Control[
    {{d, 1}, 0.01, 10, ImageSize → Tiny, Appearance → {"Labeled"}}],
   ContentSize → 180], "  ", Manipulate[
   Plot[d q^(-ϵ), {q, 0, 10}, AxesLabel → {q, "P(q)"},
    PlotRange → {0, d}, ImageSize → {120, 120}, PlotStyle → Purple],
   Column[{"Hiperbolikus árfüggvény: ",
     ToString[TraditionalForm[P[q] == d q^-ϵ]]},
    Alignment → {Center, BaseLine}, ItemSize → {17, 1}],
   Delimiter, Control[{{ϵ, 1}, 0.01, 2, ImageSize → Tiny,
     Appearance → {"Labeled"}}], Control[
    {{d, 5}, 0, 10, ImageSize → Tiny, Appearance → {"Labeled"}}]]}]
```

2. ábra

```
Manipulate[Row[{Plot[(FC / q + a + b q^c),
     {q, 1, 5}, PlotRange → {{0, 5}, {0, FC + a + b 10^c}},
     PlotLabel → "Átlagköltség függvény", AxesLabel → {q, AC},
     ImageSize → {150, 150}, PlotStyle → Purple],
    "   ", Plot[q (FC / q + a + b q^c), {q, 1, 5},
     PlotRange → {{0, 5}, {0, FC + a + b 5^(c + 1.3)}},
     PlotLabel → "Teljes költség függvény", AxesLabel → {q, TC},
     AxesOrigin → {0, 0}, ImageSize → {150, 150}, PlotStyle → Purple]}],
 Delimiter, Row[{Control[{{FC, 13}, 0, 20, ImageSize → Tiny,
      Appearance → {"Labeled"}}], "              ",
    Control[{{a, 5}, 0, 10, ImageSize → Tiny, Appearance → {"Labeled"}}]}],
 Row[{"   ", Control[{{b, 1.5}, 0, 10, ImageSize → Tiny,
      Appearance → {"Labeled"}}], "              ",
    Control[{{c, 1.5}, 0, 2, ImageSize → Tiny, Appearance → {"Labeled"}}]}],
 Delimiter, ContentSize → {350, 170}]
```

3. Versengés, a Cournot-duopólium

3.1. Időben állandó modell

1. ábra

```
Show[ContourPlot[{solCoke[[2]] - qc == 0, solPepsi[[2]] - qp == 0,
   πCoke == C1, πPepsi == C2}, {qc, 0, 4}, {qp, 0, 15},
  ContourStyle → {{Thick, Dashed, Blue}, {Thick, Dashed, Red},
    {Blue, Thick, Opacity[0.5]}, {Red, Thick, Opacity[0.5]}},
  ContourShading → False], ContourPlot[
  Evaluate[Flatten[Map[{πCoke == #, πPepsi == #} &, Range[0, 20, 4]]]],
  {qc, 0, 4}, {qp, 0, 15},
  ContourStyle → {{Blue, Opacity[0.3]}, {Red, Opacity[0.3]}},
  ContourShading → False, Contours → 10], ImageSize → {180, 180},
 FrameLabel → {Subscript[q, c], Subscript[q, p]}]
```

Interaktív kísérletek

```
Clear["Global`*"];
Manipulate[TabView[{"Profitfüggvények" → DynamicModule[
     {P, q, q1, q2, Psum, TC1, TC2, TR1, TR2, π1, π2, Pi1max, Pi2max, π1max,
      π2max, Pplot, TC1plot, TC2plot, qmax, grprofit}, P = Which[Pr === 1,
       b - d (w q1 + (1 - w) q2), Pr === 2, d (w q1 + (1 - w) q2) ^ (-ϵ)];
     Psum = Which[Pr === 1, Max[b - d q, 0], Pr === 2, d q ^ (-ϵ)];
     TC1 = q1 (FC1 / q1 + a1 + b1 q1 ^ c1);
     TC2 = q2 (FC2 / q2 + a2 + b2 q2 ^ c2);
     TR1 = P q1; TR2 = P q2;
     π1 = TR1 - TC1;
     π2 = TR2 - TC2;
     qmax = Which[Pr === 1, b / d, Pr === 2, d];
     Pi1max = Quiet[FindMaximum[π1, {{q1}, {q2}}]];
     π1max = Pi1max[[1]];
     Pi2max = Quiet[FindMaximum[π2, {{q1}, {q2}}]];
     π2max = Pi2max[[1]];
     grprofit =
      Row[{If[π1max < 0, Style["Nincs pozitív lokális maximum!"],
          Show[Plot3D[Evaluate[π1], {q1, 0, 5}, {q2, 0, 5},
            AxesLabel → {"\!\(\*SubscriptBox[\(q\), \(1\)]\)",
              "\!\(\*SubscriptBox[\(q\), \(2\)]\)",
              "\!\(\*SubscriptBox[\(π\), \(1\)]\)"},
            MeshFunctions → {#3 &}, Mesh → {{0}}, MeshStyle → {Thick, Blue},
            ImageSize → {130, 130}], Plot3D[0, {q1, 0, 5}, {q2, 0, 5},
            AxesLabel → {"\!\(\*SubscriptBox[\(q\), \(1\)]\)",
              "\!\(\*SubscriptBox[\(q\), \(2\)]\)",
              "\!\(\*SubscriptBox[\(π\), \(1\)]\)"},
            Mesh → None, PlotStyle → {Purple, Opacity[0.2]},
            PlotRange → All, ImageSize → {130, 130}],
           PlotRange → {pr * Automatic, Automatic}]], "  ",
         If[π2max < 0, Style["Nincs pozitív lokális maximum!"],
          Show[Plot3D[Evaluate[π2], {q1, 0, 5}, {q2, 0, 5},
            AxesLabel → {"\!\(\*SubscriptBox[\(q\), \(1\)]\)",
```

```
          "\!\(\*SubscriptBox[\(q\), \(2\)]\)",
          "\!\(\*SubscriptBox[\(π\), \(2\)]\)"},
        MeshFunctions → {#3 &}, Mesh → {{0}}, MeshStyle → {Thick, Red},
        ImageSize → {130, 130}], Plot3D[0, {q1, 0, 5}, {q2, 0, 5},
        AxesLabel → {"\!\(\*SubscriptBox[\(q\), \(1\)]\)",
          "\!\(\*SubscriptBox[\(q\), \(2\)]\)",
          "\!\(\*SubscriptBox[\(π\), \(1\)]\)"},
        Mesh → None, PlotStyle → {Purple, Opacity[0.2]},
        PlotRange → All, ImageSize → {130, 130}],
       PlotRange → {pr * Automatic, Automatic}]]}];
  Pplot = Plot[Psum, {q, 0, qmax}, PlotStyle → Purple, AxesLabel →
     {"Q", "P(Q)"}, PlotLabel → Text[Style["Árfüggvény:P(Q)", 10]],
    PlotRange → {{0, qmax}, {0, b}}, ImageSize → {90, 90}];
  TC1plot = Plot[TC1, {q1, 0.1, 10},
    PlotRange → {{0, 10}, {0, FC1 + a1 + b1 10^(c1 + 1.5)}},
    PlotLabel → Text[Style["TC=FC+VC", 10]],
    PlotStyle → Blue, AxesLabel → {"q", "TC"},
    AxesOrigin → {0, 0}, ImageSize → {90, 90}];
  TC2plot = Plot[TC2, {q2, 0.1, 10}, PlotStyle → Red,
    AxesOrigin → {0, 0}, ImageSize → {90, 90}];
  Column[{Row[{Show[TC1plot, TC2plot], "      ", Pplot}], grprofit}]],
"Cournot-megoldás" → DynamicModule[{P, q, q1, q2, Psum, TC1, TC2,
   TR1, TR2, π1, π2, DPi1q1, DPi2q2, SolF, qmax, Sol, grrival,
   apr, bpr, x, y, Pplot, TC1plot, TC2plot}, P = Which[Pr === 1,
    b - d (w q1 + (1 - w) q2), Pr === 2, d (w q1 + (1 - w) q2)^(-ϵ)];
  Psum = Which[Pr === 1, Max[b - d q, 0], Pr === 2, d q^(-ϵ)];
  TC1 = q1 (FC1 / q1 + a1 + b1 q1^c1);
  TC2 = q2 (FC2 / q2 + a2 + b2 q2^c2);
  TR1 = P q1; TR2 = P q2;
  π1 = TR1 - TC1;
  π2 = TR2 - TC2;
  DPi1q1 = D[π1, q1];
  DPi2q2 = D[π2, q2];
  SolF = Quiet[FindRoot[
     Evaluate[{DPi1q1 == 0, DPi2q2 == 0}], {{q1, Q1}, {q2, Q2}}]];
  qmax = Which[Pr === 1, b / d, Pr === 2, d];
  Sol =
   If[Length[SolF] < 2, {}, If[TrueQ[Length[SolF] > 2], SolF, Switch[
      Length[SolF] == 2 && Re[SolF[[1, 2]]] > 0 && Re[SolF[[2, 2]]] > 0 &&
       Im[SolF[[1, 2]]] == 0 && Im[SolF[[2, 2]]] == 0 &&
       Re[π1 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] > 0 &&
       Re[π2 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] > 0 &&
       Im[π1 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] == 0 &&
       Im[π2 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] == 0,
      True, SolF, False, {}]]];
  If[Sol ≠ {}, x = Sol[[1, 2]] * 2.5 + 1;
   y = Sol[[2, 2]] * 2.5 + 1;
   apr = π1 /. {q1 → Sol[[1, 2]], q2 → Sol[[2, 2]]};
   bpr = π2 /. {q1 → Sol[[1, 2]], q2 → Sol[[2, 2]]};, x = 10;
   y = 10;
   apr = π1 /. {q1 → x, q2 → y};
   bpr = π2 /. {q1 → x, q2 → y};];
  grrival =
   Row[{Show[If[Sol ≠ {}, ContourPlot[{DPi1q1 == 0, DPi2q2 == 0}, {q1,
          0.01, x}, {q2, 0.01, y}, ContourStyle → {{Thick, Dashed, Blue},
           {Thick, Dashed, Red}}, ContourShading → False], Graphics[]],
      ContourPlot[{π1 == apr, π2 == bpr}, {q1, 0.01, x},
       {q2, 0.01, y}, ContourStyle → {{Blue, Thick, Opacity[0.5]},
```

```
            {Red, Thick, Opacity[0.5]}}, ContourShading → False],
          ContourPlot[Evaluate[Flatten[Map[{π1 == #, π2 == #} &,
              {Range[0, apr, apr / 4], Range[0, bpr, bpr / 4]}]]]],
            {q1, 0, x}, {q2, 0, y}, ContourStyle → {{Blue, Opacity[0.3]},
              {Red, Opacity[0.3]}}, ContourShading → False],
          FrameLabel → {"Subscript[q, 1]", "Subscript[q, 2]"},
          ImageSize → {130, 130}, Frame → True], "  ",
         If[Sol ≠ {}, Column[{Text[Style["Verseny esetén ", 9]],
            Row[{Text[Style[N[apr, 5], 9]], Text[Style[" + ", 10]],
              Text[Style[N[bpr, 5], 10]], Text[Style[" = ", 9]]}],
            Text[Style[N[apr + bpr, 5] " profit ", 9]],
            Text[Style[N[P /. {q1 → Sol[[1, 2]], q2 → Sol[[2, 2]]}, 5]
                " eladási áron ", 9]], Row[{Text[Style[N[Sol[[1, 2]], 3]
                  " és ", 9]], Text[Style[N[Sol[[2, 2]], 3], 9]]}],
            Row[{Text[Style["mennyiségekkel", 9]],
              Text[Style[" realizálható. ", 9]]}]}],
          Framed@Style["Nincs Cournot-megoldás!"], 14]}];
      Pplot = Plot[Psum, {q, 0, qmax}, PlotStyle → Purple, AxesLabel →
         {"Q", "P(Q)"}, PlotLabel → Text[Style["Árfüggvény:P(Q)", 10]],
        PlotRange → {{0, qmax}, {0, b}}, ImageSize → {90, 90}];
      TC1plot = Plot[TC1, {q1, 0.1, 10},
        PlotRange → {{0, 10}, {0, FC1 + a1 + b1 10^(c1 + 1.5)}},
        PlotLabel → Text[Style["TC=FC+VC", 10]],
        PlotStyle → Blue, AxesLabel → {"q", "TC"},
        AxesOrigin → {0, 0}, ImageSize → {90, 90}];
      TC2plot = Plot[TC2, {q2, 0.1, 10}, PlotStyle → Red,
        AxesOrigin → {0, 0}, ImageSize → {90, 90}];
      Column[{Row[{Show[TC1plot, TC2plot], "      ", Pplot}], grrival}]]},
   Dynamic[tbwpos]], Row[{Style["     Időben állandó modell: ", 12, Bold],
    Style["Versengés", 12, Bold]}, Alignment → {Center}],
  Delimiter, Column[{Row[{Style["Keresleti súly: ", 10],
      Text[Style["Q=w q1+(1-w)q2", 10]]}], Control[{{w, 0.5, "w"}, 0.3, 0.7,
      ImageSize → Tiny, Appearance → {"Labeled"}}]}], Delimiter, Column[
   {Control[{{Pr, 1, "Árfüggvény: P(Q)="}, {1 → Row[{Style["b", Italic],
          " - ", Style["d", Italic], Style["Q", Italic]}],
        2 → Style["d \!\(\*SuperscriptBox[\(Q\), \(-∊\)]\)", Italic]},
      PopupMenu}]}], {{b, 90}, ControlType → None},
  {{d, 40}, ControlType → None}, {{∊, 1}, ControlType → None},
  PaneSelector[
   {1 → Row[{Row[{"b", " ", Manipulator[Dynamic[b],
          {20, 100}, ImageSize → Tiny, Appearance → "Labeled"]}],
       "        ", Row[{"d", " ", Manipulator[Dynamic[d], {0.5, 50},
          ImageSize → Tiny, Appearance → "Labeled"]}]}],
    2 → Row[{Row[{"d", " ", Manipulator[Dynamic[d], {0.5, 50},
          ImageSize → Tiny, Appearance → "Labeled"]}], "        ",
       Row[{"∊", " ", Manipulator[Dynamic[∊], {0.01, 1.5},
          ImageSize → Tiny, Appearance → "Labeled"]}]}]}, Dynamic[Pr]],
  Delimiter, Row[{Text[Style["Első vállalat költségfüggvénye:", 10]],
    "   ", Text[Style["Második vállalat költségfüggvénye:", 10]]}],
  Row[{Text[Style[
      "\!\(\*SubscriptBox[\(TC\), \(1\)]\)=\!\(\*SubscriptBox[\(FC\),
        \(1\)]\)+\!\(\*SubscriptBox[\(VC\), \(1\)]\)", 10]],
    "                          ", Text[Style[
      "\!\(\*SubscriptBox[\(TC\), \(2\)]\)=\!\(\*SubscriptBox[\(FC\),
        \(2\)]\)+\!\(\*SubscriptBox[\(VC\), \(2\)]\)", 10]]}],
  Row[{Control[{{FC1, 13, "Subscript[FC, 1]"}, 0, 15, ImageSize → Tiny,
      Appearance → {"Labeled"}}], "  ",
    Control[{{FC2, 13, "Subscript[FC, 2]"}, 0, 15, ImageSize → Tiny,
```

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      Appearance → {"Labeled"}}]}], Row[{Text[Style[
      "\!\(\*SubscriptBox[\(VC\), \(1\)]\)=\!\(\*SubscriptBox[\(a\),
        \(1\)]\)\!\(\*SubscriptBox[\(q\),
        \(1\)]\)+\!\(\*SubscriptBox[\(b\),
        \(1\)]\)\!\(\*SuperscriptBox[SubscriptBox[\(q\),
        \(1\)], \(\*SubscriptBox[\(c\), \(1\)] + 1\)]\)",
      10]], "                  ", Text[Style[
      "\!\(\*SubscriptBox[\(VC\), \(2\)]\)=\!\(\*SubscriptBox[\(a\),
        \(2\)]\)\!\(\*SubscriptBox[\(q\),
        \(2\)]\)+\!\(\*SubscriptBox[\(b\),
        \(2\)]\)\!\(\*SuperscriptBox[SubscriptBox[\(q\),
        \(2\)], \(\*SubscriptBox[\(c\), \(2\)] + 1\)]\)", 10]]}],
  Row[{Control[{{a1, 5, "Subscript[a, 1]"}, 0, 5, ImageSize → Tiny,
      Appearance → {"Labeled"}}],
    "    ", Control[{{a2, 5, "Subscript[a, 2]"}, 0, 5,
      ImageSize → Tiny, Appearance → {"Labeled"}}]}],
  Row[{Control[{{b1, 1.5, "Subscript[b, 1]"}, 0, 5,
      ImageSize → Tiny, Appearance → {"Labeled"}}],
    "    ", Control[{{b2, 1.5, "Subscript[b, 2]"}, 0, 5,
      ImageSize → Tiny, Appearance → {"Labeled"}}]}],
  Row[{Control[{{c1, 0.2, "Subscript[c, 1]"}, 0, 1,
      ImageSize → Tiny, Appearance → {"Labeled"}}],
    "    ", Control[{{c2, 0.2, "Subscript[c, 2]"}, 0, 1,
      ImageSize → Tiny, Appearance → {"Labeled"}}]}],
  Delimiter, Row[{Control[{{pr, 0, "Full PlotRange"}, {0, 1}}],
    "                      ",
    Control[{{Sols, False, "Kezdeti mennyiségek"},
      {True, False}, ControlType → Checkbox}]}],
  {{Q1, 10, "\!\(\*SubscriptBox[\(q\), \(1\)]\)"},
   ControlType → None},
  {{Q2, 10, "\!\(\*SubscriptBox[\(q\), \(2\)]\)"},
   ControlType → None},
  PaneSelector[
   {True → Row[{Row[{"\!\(\*SubscriptBox[\(q\), \(1\)]\)", " ",
          Manipulator[Dynamic[Q1], {0.01, 20}, ImageSize → Tiny,
           Appearance → "Labeled"]}], "        ", Row[
         {"\!\(\*SubscriptBox[\(q\), \(2\)]\)", " ", Manipulator[Dynamic[
             Q2], {0.01, 20}, ImageSize → Tiny, Appearance → "Labeled"]}]}],
    False → Row[{" "}]}, Dynamic[Sols]], SaveDefinitions → True]
```

3.2. Időben diszkrét modell

2. ábra

```
Graphics3D[{sol /. {i_, u_, v_} → {Hue[i / NN], Point[{i, u, v}]},
   {Opacity[0.3], Thickness[0.007], Line[sol /. {i_, u_, v_} → {i, u, v}]}},
  ImageSize → {300, 200}, Axes → True,
  AxesLabel → {t, Subscript[q, c], Subscript[q, p]}, BoxRatios → {2, 1, 1}]
```

Interaktív kísérletek

```
Clear["Global`*"];
Manipulate[DynamicModule[
  {P, q, q1, q2, Psum, TC1, TC2, TR1, TR2, π1, π2, Sol, SolF, Dπ1q1, Dπ2q2,
```

```
    qmax, sol, grrival, var, f, IC, NN, Pplot, TC1plot, TC2plot}, P = Which[
     Pr === 1, b - d (w q1 + (1 - w) q2), Pr === 2, d (w q1 + (1 - w) q2) ^ (-ε)];
   Psum = Which[Pr === 1, Max[b - d q, 0], Pr === 2, d q^(-ε)];
   TC1 = q1 (FC1 / q1 + a1 + b1 q1^c1);
   TC2 = q2 (FC2 / q2 + a2 + b2 q2^c2);
   TR1 = P q1; TR2 = P q2;
   π1 = TR1 - TC1;
   π2 = TR2 - TC2;
   qmax = Which[Pr === 1, b / d, Pr === 2, d];
   NN = nn;
   Dπ1q1 = D[π1, q1];
   Dπ2q2 = D[π2, q2];
   SolF =
    Quiet[FindRoot[Evaluate[{Dπ1q1 == 0, Dπ2q2 == 0}], {{q1, 1}, {q2, 1}}]];
   Sol = If[Length[SolF] < 2, {}, If[TrueQ[Length[SolF] > 2],
       SolF, Switch[Length[SolF] == 2 && Re[SolF[[1, 2]]] > 0 &&
          Re[SolF[[2, 2]]] > 0 && Im[SolF[[1, 2]]] == 0 && Im[SolF[[2, 2]]] == 0 &&
          Re[π1 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] > 0 &&
          Re[π2 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] > 0 &&
          Im[π1 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] == 0 &&
          Im[π2 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] == 0,
        True, SolF, False, {}]]];
   var = {q1, q2};
   f[{t_, q11_, q22_}] := {t + 1, Re[q1 /.
        (FindRoot[(Evaluate[D[π1, q1]] /. q2 → q22) == 0, {q1, q11}])], Re[
      q2 /. (FindRoot[(Evaluate[D[π2, q2]] /. q1 → q11) == 0, {q2, q22}])]};
   IC = Flatten[Table[{0, i, j}, {i, Q1min, Q1max, 1},
      {j, Q2min, Q2max, 1}], 1];
   sol = Quiet[Map[NestList[f, #, NN] &, IC]];
   grrival = If[Sol ≠ {},
     Row[{Graphics3D[{sol /. {i_?NumericQ, u_?NumericQ, v_?NumericQ} →
            {Hue[i / NN], Point[{i, u, v}]}, {Opacity[0.3],
           Thickness[0.007], Line[sol /. {i_, u_, v_} → {i, u, v}]}},
         ImageSize → {110, 110}, Axes → True,
         AxesLabel → {"t", "\!\(\*SubscriptBox[\(q\), \(1\)]\)",
           "\!\(\*SubscriptBox[\(q\), \(2\)]\)"}, BoxRatios → {2, 1, 1}],
        "  ", Column[{Text[Style["Egyensúlyi helyzet:", 10]],
          Row[{"(", Text[Style[Sol[[1, 2]], 10]], ",",
            Text[Style[Sol[[2, 2]], 10]], ")"}], Row[
           {Text[Style["\!\(\*SubscriptBox[\(π\), \(1\)]\)= ", 10]], Text[
             Style[π1 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}, 10]]}], Row[
           {Text[Style["\!\(\*SubscriptBox[\(π\), \(2\)]\)= ", 10]], Text[
             Style[π2 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}, 10]]}]}]}],
     Text[Style["Nincs Cournot-megoldás!"]]];
   Pplot = Plot[Psum, {q, 0, qmax}, PlotStyle → Purple, AxesLabel →
      {"Q", "P(Q)"}, PlotLabel → Text[Style["Árfüggvény:P(Q)", 10]],
     PlotRange → {{0, qmax}, {0, b}}, ImageSize → {80, 80}];
   TC1plot = Plot[TC1, {q1, 0.1, 10},
     PlotRange → {{0, 10}, {0, FC1 + a1 + b1 10^(c1 + 1.5)}},
     PlotLabel → Text[Style["TC=FC+VC", 10]], PlotStyle → Blue,
     AxesLabel → {"q", "TC"}, AxesOrigin → {0, 0}, ImageSize → {80, 80}];
   TC2plot = Plot[TC2, {q2, 0.1, 10}, PlotStyle → Red,
     AxesOrigin → {0, 0}, ImageSize → {80, 80}];
   Column[{Row[{Show[TC1plot, TC2plot], "       ", Pplot}], grrival}]],
  Row[{Style["     Időben diszkrét modell: ", 12, Bold],
    Style["Versengés", 12, Bold]}, Alignment → {Center}],
  Delimiter, Column[{Row[{Style["Keresleti súly: ", 10],
      Text[Style["Q=w Subscript[q, 1]+(1-w)Subscript[q, 2]", 10]]}],
```

```
   Control[{{w, 0.5, "w"}, 0.3, 0.7, ImageSize → Tiny,
     Appearance → {"Labeled"}}]}], Delimiter, Column[
  {Control[{{Pr, 1, "Árfüggvény: P(Q)="}, {1 → Row[{Style["b", Italic],
          " - ", Style["d", Italic], Style["Q", Italic]}],
      2 → Style["d \!\(\*SuperscriptBox[\(Q\), \(-ϵ\)]\)", Italic]},
     PopupMenu}]}], {{b, 90}, ControlType → None},
 {{d, 40}, ControlType → None}, {{ϵ, 1}, ControlType → None},
 PaneSelector[
  {1 → Row[{Row[{"b", " ", Manipulator[Dynamic[b],
          {20, 100}, ImageSize → Tiny, Appearance → "Labeled"]}],
      "           ", Row[{"d", " ", Manipulator[Dynamic[d],
          {0.01, 40}, ImageSize → Tiny, Appearance → "Labeled"]}]}],
   2 → Row[{Row[{"d", " ", Manipulator[Dynamic[d], {0.01, 50},
          ImageSize → Tiny, Appearance → "Labeled"]}], "          ",
      Row[{"ϵ", " ", Manipulator[Dynamic[ϵ], {0.01, 1.5},
          ImageSize → Tiny, Appearance → "Labeled"]}]}]}, Dynamic[Pr]],
 Delimiter, Row[{Text[Style["Első vállalat költségfüggvénye:", 10]],
   "   ", Text[Style["Második vállalat költségfüggvénye:", 10]]}],
 Row[{Text[Style[
     "\!\(\*SubscriptBox[\(TC\), \(1\)]\)=\!\(\*SubscriptBox[\(FC\),
       \(1\)]\)+\!\(\*SubscriptBox[\(VC\), \(1\)]\)", 10]],
   "                              ", Text[Style[
     "\!\(\*SubscriptBox[\(TC\), \(2\)]\)=\!\(\*SubscriptBox[\(FC\),
       \(2\)]\)+\!\(\*SubscriptBox[\(VC\), \(2\)]\)", 10]]}],
 Row[{Control[{{FC1, 13, "Subscript[FC, 1]"}, 0, 15, ImageSize → Tiny,
     Appearance → {"Labeled"}}], "     ",
   Control[{{FC2, 13, "Subscript[FC, 2]"}, 0, 15, ImageSize → Tiny,
     Appearance → {"Labeled"}}]}], Row[{Text[Style[
     "\!\(\*SubscriptBox[\(VC\), \(1\)]\)=\!\(\*SubscriptBox[\(a\),
       \(1\)]\)\!\(\*SubscriptBox[\(q\),
       \(1\)]\)+\!\(\*SubscriptBox[\(b\),
       \(1\)]\)\!\(\*SuperscriptBox[SubscriptBox[\(q\),
       \(1\)], \(\*SubscriptBox[\(c\), \(1\)] + 1\)]\)", 10]],
   "                        ", Text[Style[
     "\!\(\*SubscriptBox[\(VC\), \(2\)]\)=\!\(\*SubscriptBox[\(a\),
       \(2\)]\)\!\(\*SubscriptBox[\(q\),
       \(2\)]\)+\!\(\*SubscriptBox[\(b\),
       \(2\)]\)\!\(\*SuperscriptBox[SubscriptBox[\(q\),
       \(2\)], \(\*SubscriptBox[\(c\), \(2\)] + 1\)]\)", 10]]}],
 Row[{Control[{{a1, 5, "Subscript[a, 1]"}, 0, 10, ImageSize → Tiny,
     Appearance → {"Labeled"}}],
   "        ", Control[{{a2, 5, "Subscript[a, 2]"}, 0, 10,
     ImageSize → Tiny, Appearance → {"Labeled"}}]}],
 Row[{Control[{{b1, 1.5, "Subscript[b, 1]"}, 0, 5,
     ImageSize → Tiny, Appearance → {"Labeled"}}],
   "        ", Control[{{b2, 1.5, "Subscript[b, 2]"}, 0, 5,
     ImageSize → Tiny, Appearance → {"Labeled"}}]}],
 Row[{Control[{{c1, 0.2, "Subscript[c, 1]"}, 0, 1,
     ImageSize → Tiny, Appearance → {"Labeled"}}],
   "        ", Control[{{c2, 0.2, "Subscript[c, 2]"}, 0, 1,
     ImageSize → Tiny, Appearance → {"Labeled"}}]}],
 Delimiter, Control[{{nn, 5, "Iteráció"}, 3, 10, 1,
   ImageSize → Tiny,
   Appearance → {"Labeled"}}],
 Delimiter, Style["Kezdeti értékek:",
  9],
 Row[{Control[{{Q1min, 1, "Subscript[q, 1,min]"}, 1,
     5, 1, ImageSize → Tiny, Appearance → {"Labeled"}}],
```

```
  " ", Control[{{Q2min, 1, "Subscript[q, 2,min]"}, 1, 5, 1,
    ImageSize → Tiny, Appearance → {"Labeled"}}]}],
Row[{Control[{{Q1max, 5, "Subscript[q, 1,max]"}, Q1min + 1,
    9, 1, ImageSize → Tiny, Appearance → {"Labeled"}}],
  Control[{{Q2max, 5, "Subscript[q, 2,max]"}, Q2min + 1, 9,
    1, ImageSize → Tiny, Appearance → {"Labeled"}}]}],
SaveDefinitions → True]
```

3.3. Időben folytonos modell

3. ábra

```
Row[
 {Show[StreamPlot[Evaluate[Rhs2D /. {k → K[[1]]}], {qC, 0, 10}, {qP, 0, 10},
    FrameLabel → {Subscript[q, c], Subscript[q, p]}],
   ParametricPlot[Evaluate[sol[t][[1, 1]]], {t, 0, 4}, PlotStyle → Red],
   ImageSize → {150, 150}],
  Show[StreamPlot[Evaluate[Rhs2D /. {k → K[[2]]}], {qC, 0, 10},
    {qP, 0, 10}, FrameLabel → {Subscript[q, c], Subscript[q, p]}],
   ParametricPlot[Evaluate[sol[t][[2, 1]]], {t, 0, 4}, PlotStyle → Red],
   ImageSize → {150, 150}]}]
```

Interaktív kísérletek

```
Clear["Global`*"];
Manipulate[
 DynamicModule[{P, q, q1, q2, Psum, TC1, TC2, TR1, TR2, π1, π2, Sol,
   SolF, Dπ1q1, Dπ2q2, qmax, solcont, grrival, var, fcont, ICcont,
   Pplot, TC1plot, TC2plot}, P = Which[Pr === 1,
    b - d (w q1 + (1 - w) q2), Pr === 2, d (w q1 + (1 - w) q2) ^ (-ϵ)];
  Psum = Which[Pr === 1, Max[b - d q, 0], Pr === 2, d q ^ (-ϵ)];
  TC1 = q1 (FC1 / q1 + a1 + b1 q1 ^ c1);
  TC2 = q2 (FC2 / q2 + a2 + b2 q2 ^ c2);
  TR1 = P q1; TR2 = P q2;
  π1 = TR1 - TC1;
  π2 = TR2 - TC2;
  var = {q1, q2};
  qmax = Which[Pr === 1, b / d, Pr === 2, d];
  Dπ1q1 = D[π1, q1];
  Dπ2q2 = D[π2, q2];
  SolF =
   Quiet[FindRoot[Evaluate[{Dπ1q1 == 0, Dπ2q2 == 0}], {{q1, 1}, {q2, 1}}]];
  Sol = If[Length[SolF] < 2, {}, If[TrueQ[Length[SolF] > 2],
     SolF, Switch[Length[SolF] == 2 && Re[SolF[[1, 2]]] > 0 &&
       Re[SolF[[2, 2]]] > 0 && Im[SolF[[1, 2]]] == 0 && Im[SolF[[2, 2]]] == 0 &&
       Re[π1 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] > 0 &&
       Re[π2 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] > 0 &&
       Im[π1 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] == 0 &&
       Im[π2 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}] == 0,
      True, SolF, False, {}]]];
  fcont[{q11_, q22_}] := {k1 (Re[q1 /. (FindRoot[
           (Evaluate[D[π1, q1]] /. q2 → q22) == 0, {q1, q11}])] - q11),
    k2 (Re[q2 /. (FindRoot[(Evaluate[D[π2, q2]] /. q1 → q11) == 0,
           {q2, q22}])] - q22)};
```

```
 ICcont = If[Sol == {}, {}, Flatten[Table[{i, j},
      {i, Max[0.1, Sol[[1, 2]] - Q1max], Sol[[1, 2]] + Q1max, 1},
      {j, Max[0.1, Sol[[2, 2]] - Q2max], Sol[[2, 2]] + Q2max, 1}], 1]];
 solcont = Quiet[Map[NestList[fcont, #, 1] &, ICcont]];
 grrival =
  If[Sol == {}, Framed@Style["Nincs Cournot-megoldás!", 12], Row[
    {Show[ListStreamPlot[solcont, ImageSize → 130, StreamStyle → Purple,
       FrameLabel → {"\!\(\*SubscriptBox[\(q\), \(1\)]\)",
         "\!\(\*SubscriptBox[\(q\), \(2\)]\)"}],
      ContourPlot[Evaluate[{D[π1, q1], D[π2, q2]}],
       {q1, 0, Sol[[1, 2]] + Q1max}, {q2, 0, Sol[[2, 2]] + Q2max},
       Contours → {{0}, {0}}, ContourStyle → {Blue, Red}]],
     "  ", Column[{Text[Style["Egyensúlyi helyzet:", 10]],
       Row[{"(", Text[Style[Sol[[1, 2]], 10]],
         ",", Text[Style[Sol[[2, 2]], 10]], ")"}],
       Row[{Text[Style["\!\(\*SubscriptBox[\(π\), \(1\)]\)= ", 10]],
         Text[Style[π1 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]}, 10]]}],
       Row[{Text[Style["\!\(\*SubscriptBox[\(π\), \(2\)]\)= ", 10]],
         Text[Style[π2 /. {q1 → SolF[[1, 2]], q2 → SolF[[2, 2]]},
           10]]}]}]}]];
 Pplot = Plot[Psum, {q, 0, qmax}, PlotStyle → Purple, AxesLabel →
     {"Q", "P(Q)"}, PlotLabel → Text[Style["Árfüggvény:P(Q)", 10]],
   PlotRange → {{0, qmax}, {0, b}}, ImageSize → {100, 100}];
 TC1plot = Plot[TC1, {q1, 0.1, 10},
   PlotRange → {{0, 10}, {0, FC1 + a1 + b1 10^(c1 + 1.5)}},
   PlotLabel → Text[Style["TC=FC+VC", 10]], PlotStyle → Blue,
   AxesLabel → {"q", "TC"}, AxesOrigin → {0, 0}, ImageSize → {100, 100}];
 TC2plot = Plot[TC2, {q2, 0.1, 10}, PlotStyle → Red,
   AxesOrigin → {0, 0}, ImageSize → {100, 100}];
 Column[{Row[{Show[TC1plot, TC2plot], "      ", Pplot}], grrival}]],
Row[{Style["     Időben folytonos modell: ", 12, Bold],
  Style["Versengés", 12, Bold]}, Alignment → {Center}],
Delimiter, Column[{Row[{Style["Keresleti súly: ", 10],
    Text[Style["Q=w q1+(1-w)q2", 10]]}], Control[{{w, 0.5, "w"}, 0.3, 0.7,
    ImageSize → Tiny, Appearance → {"Labeled"}}]}], Delimiter, Column[
 {Control[{{Pr, 1, "Árfüggvény: P(Q)="}, {1 → Row[{Style["b", Italic],
         " - ", Style["d", Italic], Style["Q", Italic]}],
      2 → Style["d \!\(\*SuperscriptBox[\(Q\), \(-ϵ\)]\)", Italic]},
    PopupMenu}]}], {{b, 90}, ControlType → None},
{{d, 40}, ControlType → None}, {{ϵ, 1}, ControlType → None},
PaneSelector[
 {1 → Row[{Row[{"b", " ", Manipulator[Dynamic[b],
         {20, 100}, ImageSize → Tiny, Appearance → "Labeled"]}],
     "         ", Row[{"d", " ", Manipulator[Dynamic[d],
         {0.01, 40}, ImageSize → Tiny, Appearance → "Labeled"]}]}],
  2 → Row[{Row[{"d", " ", Manipulator[Dynamic[d], {0.01, 50},
         ImageSize → Tiny, Appearance → "Labeled"]}], "        ",
     Row[{"ϵ", " ", Manipulator[Dynamic[ϵ], {0.01, 1.5},
         ImageSize → Tiny, Appearance → "Labeled"]}]}]}, Dynamic[Pr]],
Delimiter, Row[{Text[Style["Első vállalat költségfüggvénye:", 10]],
  "   ", Text[Style["Második vállalat költségfüggvénye:", 10]]}],
Row[{Text[Style[
    "\!\(\*SubscriptBox[\(TC\), \(1\)]\)=\!\(\*SubscriptBox[\(FC\),
      \(1\)]\)+\!\(\*SubscriptBox[\(VC\), \(1\)]\)", 10]],
  "                                ", Text[Style[
    "\!\(\*SubscriptBox[\(TC\), \(2\)]\)=\!\(\*SubscriptBox[\(FC\),
      \(2\)]\)+\!\(\*SubscriptBox[\(VC\), \(2\)]\)", 10]]}],
Row[{Control[{{FC1, 13, "Subscript[FC, 1]"}, 0, 15, ImageSize → Tiny,
```

```
    Appearance → {"Labeled"}}], "  ",
  Control[{{FC2, 13, "Subscript[FC, 2]"}, 0, 15, ImageSize → Tiny,
    Appearance → {"Labeled"}}]}], Row[{Text[Style[
    "\!\(\*SubscriptBox[\(VC\), \(1\)]\)=\!\(\*SubscriptBox[\(a\),
      \(1\)]\)\!\(\*SubscriptBox[\(q\),
      \(1\)]\)+\!\(\*SubscriptBox[\(b\),
      \(1\)]\)\!\(\*SuperscriptBox[SubscriptBox[\(q\),
      \(1\)], \(\*SubscriptBox[\(c\), \(1\)] + 1\)]\)",
    10]], "                  ", Text[Style[
    "\!\(\*SubscriptBox[\(VC\), \(2\)]\)=\!\(\*SubscriptBox[\(a\),
      \(2\)]\)\!\(\*SubscriptBox[\(q\),
      \(2\)]\)+\!\(\*SubscriptBox[\(b\),
      \(2\)]\)\!\(\*SuperscriptBox[SubscriptBox[\(q\),
      \(2\)], \(\*SubscriptBox[\(c\), \(2\)] + 1\)]\)", 10]]}],
Row[{Control[{{a1, 5, "Subscript[a, 1]"}, 0, 10, ImageSize → Tiny,
    Appearance → {"Labeled"}}],
  "    ", Control[{{a2, 5, "Subscript[a, 2]"}, 0, 10,
    ImageSize → Tiny, Appearance → {"Labeled"}}]}],
Row[{Control[{{b1, 1.5, "Subscript[b, 1]"}, 0, 5,
    ImageSize → Tiny, Appearance → {"Labeled"}}],
  "    ", Control[{{b2, 1.5, "Subscript[b, 2]"}, 0, 5,
    ImageSize → Tiny, Appearance → {"Labeled"}}]}],
Row[{Control[{{c1, 0.2, "Subscript[c, 1]"}, 0, 1,
    ImageSize → Tiny, Appearance → {"Labeled"}}],
  "    ", Control[{{c2, 0.2, "Subscript[c, 2]"}, 0, 1,
    ImageSize → Tiny, Appearance → {"Labeled"}}]}],
Delimiter, Text[Style["Reagálás gyorsasága:",
  9]],
Row[{Control[{{k1, 0.5, "\!\(\*SubscriptBox[\(k\), \(1\)]\)"},
    0.01, 1, ImageSize → Tiny, Appearance → {"Labeled"}}], "    ",
  Control[{{k2, 0.5, "\!\(\*SubscriptBox[\(k\), \(2\)]\)"},
    0.01, 1, ImageSize → Tiny, Appearance → {"Labeled"}}]}],
Delimiter, Style["Megoldástól való távolság:",
 9],
Row[{Control[{{Q1max, 5, "\!\(\*SubscriptBox[\(q\), \(1\)]\)"},
    3, 20, 1, ImageSize → Tiny, Appearance → {"Labeled"}}], "    ",
  Control[{{Q2max, 5, "\!\(\*SubscriptBox[\(q\), \(2\)]\)"},
    3, 20, 1, ImageSize → Tiny, Appearance → {"Labeled"}}]}],
SaveDefinitions → True]
```

4. Kartell két vállalat esetén

4.1. Időben állandó modell

Interaktív kísérletek

Az interaktív kísérlet programkódja a 3.1 interaktív kísérletének programkódjához hasonló. Ebben azesetben viszont a két vállalat által közösen realizálható profitot maximalizáljuk:

```
πkozos = π1 + π2
```

4.3. Időben folytonos modell

1. ábra

```
Show[StreamPlot[Evaluate[Rhs2D], {qC, 14, 16},
   {qP, -14, -16}, FrameLabel → {Subscript[q, c], Subscript[q, p]}],
 ParametricPlot[Evaluate[sol[t]], {t, 0, 500}, PlotStyle → Red],
 ImageSize → {200, 200}]
```

Interaktív kísérletek

Az interaktív kísérlet programkódja a 3.3 interaktív kísérletének programkódjához hasonló. Ebben azesetben viszont a két vállalat által közösen realizálható profitot maximalizáljuk:

```
πkozos = π1 + π2
```

5. Korlátozott kibocsátású Cournot-duopólium

5.1. Korlátozott kibocsátású modell

1. ábra

```
Plot[-Log[1 - q], {q, 0, 1}, AxesLabel → {q / u, TC},
 PlotRange → {0, 10}, ImageSize → {150, 150}, PlotStyle → Purple]
```

2. ábra

```
Manipulate[DynamicModule[{Sol, a, b},
   Sol = Solve[{q1 == sol1[[2, 1, 2]], q2 == sol2[[2, 1, 2]]}, {q1, q2}][[2]];
   a = π1 /. Sol;
   b = π2 /. Sol;
   Show[
    ContourPlot[{π1 == a, π2 == b}, {q1, 0, u / 2}, {q2, 0, v / 2}, ContourStyle →
       {{Thick, Blue}, {Thick, Red}, {Blue, Thick, Opacity[0.5]},
        {Red, Thick, Opacity[0.5]}}, ContourShading → False,
     FrameLabel → {"Subscript[q, 1]", "Subscript[q, 2]"}], ContourPlot[
     Evaluate[Flatten[Map[{π1 == #, π2 == #} &, Range[0, a, 0.1]]]],
     {q1, 0, u / 2}, {q2, 0, v / 2},
     ContourStyle → {{Blue, Opacity[0.3]}, {Red, Opacity[0.3]}},
     ContourShading → False, Contours → 10], ImageSize → {150, 150}]],
 {{u, 20}, ControlType → None}, {{v, 10}, ControlType → None},
 Row[{Row[{"u", " ", Manipulator[Dynamic[u], {5, 20},
        ImageSize → Tiny, Appearance → "Labeled"]}], "",
   Row[{"v", " ", Manipulator[Dynamic[v], {5, 20}, ImageSize → Tiny,
```

```
      Appearance → "Labeled"]}]}], SaveDefinitions → True]
```

◻ 3. ábra

```
Show[ContourPlot[Abs[Ev1], {u, 0, 2}, {v, 0, 2},
  ImageSize → {200, 200}, FrameLabel → {u, v}], ContourPlot[
  Abs[Ev1] == 1, {u, 0, 2}, {v, 0, 2}, ContourStyle → {Black, Thick},
  ImageSize → {200, 200}, FrameLabel → {u, v}]]
```

◻ 4. ábra

```
Graphics3D[
 {sol /. {i_Real, x_Real, y_Real} → {Hue[i / NN], Point[{i, x, y}]},
  {Opacity[0.3], Thickness[0.007], Line[sol /. {i_, x_, y_} → {i, x, y}]}},
 ImageSize → {200, 200}, Axes → True,
 AxesLabel → {t, Subscript[q, 1], Subscript[q, 2]},
 BoxRatios → Automatic, PlotRange → {{0, 5}, {0, 4}, {0, 4}}]
```

◻ 5. ábra

```
Graphics3D[
 {sol /. {i_Real, x_Real, y_Real} → {Hue[i / NN], Point[{i, x, y}]},
  {Opacity[0.3], Thickness[0.007],
   Line[sol /. {i_Real, x_Real, y_Real} → {i, x, y}]}},
 ImageSize → {200, 200}, Axes → True,
 AxesLabel → {t, Subscript[q, 1], Subscript[q, 2]},
 PlotRange → {{0, 5}, {0, 10}, {0, 5}}, BoxRatios → {2, 1, 1}]
```

◻ Interaktív kísérletek: Állandó stratégia

Az interaktív kísérlet programkódja a 3.1. interaktív kísérletének programkódja alapján készült. A teljes költségfüggvények ebben az esetben:

```
TC1 = -Log[1 - q1 / u]; TC2 = -Log[1 - q2 / v];
```

◻ Interaktív kísérletek: Diszkrét stratégia

Az interaktív kísérlet programkódja a 3.2. interaktív kísérletének programkódja alapján készült. A teljes költségfüggvények ebben a modellben:

```
TC1 = -Log[1 - q1 / u]; TC2 = -Log[1 - q2 / v];
```