



# Computer -Aided modeling with Mathematica 6

## **Compact course**

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# Starting out in Mathematica

## Introduction

Introductory tour <u>The basic concepts of Mathematica</u>: Front-End functions, Kernel, Help <u>Typesetting:</u> cells, formula forms <u>Basic operations:</u> Numeric and symbolic operations, variables, algebraic manipulations <u>Lists, vectors and arrays</u> <u>Basic plot statements</u> <u>Solving equations</u>

## Notebook operations I

Cell styles, stylesheets

## Animations

Simple tools for animations

Examples

Zooming Animating trigonometric series Lissajous curves

## Fundamentals of the structures in Mathematica

<u>Setting, Rules, Functions</u> <u>Data structures, head operations and parameter-type check</u>

## Graphics fundamentals

Steps of generating plots and graphics Summary of plot functions

### Plots in 2D

<u>Plotting:functions</u> <u>Plotting lists in 2D: ListPlot</u> <u>Parametric curves: ParametricPlot</u>

Examples

Animation of moving point in 2D Inverse function Animate the definition of Sin[x] Animating a pendulum

### Plots in 3D

Functions of two variables, scalarfields  $(F : \mathbb{R}^2 \to \mathbb{R})$ : Plot3D, ContourPlot, DensityPlot Special new features for 3D Graphics in Version 6. <u>3D Parametric curves and surfaces</u>: ParametricPlot3D and its versions Contoursurfaces of scalarfields  $V : \mathbb{R}^3 \to \mathbb{R}$ 

Examples

<u>Plot functions in different coordinate systems</u> <u>Visualization of moving points in 3D</u>

### Graphics summary

Graphics structures in Version 6. Graphics Structures in Version 5 Graphics conversions in Vesrsin 5

#### **Examples**

Simple volume rendering: plot and color the points of a volume Visualization of series expansions, spectrum animation

## **Basic applications in Mathematics**

#### Data manipulations

Nonlinear transformations on planar datasets

Graphical study of 2D experimental data: transformations, special plots

### Vectors, Matrices, Linear Algebra

Linear Algebra: vectors, matrices, transformations, eigenvalues, eigenvectors, etc.

### Calculus

Calculus summary

Applications and examples in 1D Calculus

Tangent line and secant lines Animation of the tangent lines Animation of trigonometric series Graphical interpretations of derivative: zooming Investigation of functions Taylor polynomials

#### Applications in 2D-3D Calculus

Calculus methods: partial derivatives, ..., maxima and minima Tangent planes and normal vectors of surfaces Tangent vectors and normal planes of curves Moving points in 2D Moving points in 3D Maxima and minima: the numerical and visual point of view Constrained extrema

#### Curve Fitting

Curve fitting

### Complex numbers, complex functions

Summary and examples

#### Differential equations I.

ODE summary Modeling with 1D ODE's 2D equations: vector fields, solutions, trajectories Trajectory animation and manipulation in 2D (using only built in functions) Trajectory animation in 3D (Lorenz system)

### Advanced programming in Mathematica

#### List programming I: Structure operations

Structure operations; Apply rules to lists

#### List programming II

Rotating lists (RotateLeft, RotateRight)

Examples

The midpoint rule Moving average of data and analogous problems Simple image processing

ListConvolve, ListCorrelate

#### Recursion, iteration

Summary and basic examples

Examples: recursion vs. nesting <u>Factorial</u> <u>Continued fractions</u>

Simple numerical algorithms Fixedpoints of mappings <u>Newton iterations</u> <u>Picard iteration</u> <u>Methods to approximate zeros of functions</u> Riemann sums

### Programming paradigms in Mathematica: a systematic treatment

<u>Procedural programming</u> <u>Functional programming</u> Rule- and patt<u>ern-based programming</u>

### Nontrivial applications

#### Addons' to list programming: some string manipulations

Examples <u>Morse</u> <u>A coding-decoding system</u>

### Graphical programming I: Advanced visualization

 Transformations on graphical (Graphics and Graphics3D) objects

 Transformations on Graphics objects (2D)

 Transformations on Graphics3D objects (3D)

 Replace anything by anything

Numerical data from graphical objects <u>Points of contour lines of planar scalar fields</u> <u>Normal vectors to a graphically given surface</u>

Advanced visualization problems (graphics from numerical data) Volume rendering, slicing; coloring the discrete space by scalar fields Volume rendering, slicing; coloring the continuous space by scalar fields Visualizing parametric curves given in lists Vector field at given contour lines in 2D Vector field at given surfaces in 3D

#### Graphical programming II: Advanced substitutions, Iterative forms

Simple iterative constructions

<u>Iterate simple substitutions (generate trees)</u> <u>Iterate the substitution of patterns</u> <u>Simple selfsimilar objects</u>

Some other examples Iterations using the midpoint rule Generate trees Some more tree-like structures Sierpinsky triangles, Koch curves Sierpinsky attractor

#### Difference equations, finite differences

Difference equations: definition, solution Cobweb diagram Logistic mapping, bifurcation diagram Solve and visualize planar difference equations Partial difference equations (a procedural way of solution) Application of list rotation to finite differences Disretization of PDE's: more examples

#### Differential equations: advanced problems

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Technical tools
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<u>General solver: ODESolve</u> <u>Visualization of solutions given in lists: Euler's method</u>

Qualitative methods <u>Stability: Liapunov method</u> <u>The phasemap method</u> <u>ODE's with Dirac delta</u> <u>Poincare maps</u>

#### Extra

#### Writing Modules and Packages

Package design, a general overview

Examples

Variableless mappings from expressions: <u>EulerDSolve</u> General program usig variable names: <u>ODESolve</u> Handling options: <u>Colored ParametricPlot</u>

#### Additional topics. Notebook operations II

Options, option inspector, simple stylesheet development Data export, import

Advanced style operations: automatic numbering, hyperlinks,... Export, import: HTML, XML, MathML, TeX

### **Exercises**

Basic exercises 1D Calculus Data handling, fitting Linear Algebra 3D Claculus, Lines and Surfaces 1D differential equations Oscillator equations Planar differential equations Programming exercises Advanced programming exercises