# Computer -Aided modeling <br> with Mathematica 6 <br> <br> Compact course 

 <br> <br> Compact course}

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Starting out in Mathematica

## Introduction

Introductory tour
The basic concepts of Mathematica: Front-End functions, Kernel, Help Typesetting: cells, formula forms
Basic operations: Numeric and symbolic operations, variables, algebraic manipulations
Lists, vectors and arrays
Basic plot statements
Solving equations

## Notebook operations I

Cell styles, stylesheets
$\square$ Animations
Simple tools for animations
Examples
Zooming
Animating trigonometric series
Lissajous curves

## Fundamentals of the structures in Mathematica

Setting, Rules, Functions
Data structures, head operations and parameter-type check

## $\square$ Graphics fundamentals

Steps of generating plots and graphics
Summary of plot functions

## Plots in 2D

Plotting:functions
Plotting lists in 2D: ListPlot
Parametric curves: ParametricPlot
Examples
Animation of moving point in 2D
Inverse function
Animate the definition of $\operatorname{Sin}[x]$
Animating a pendulum

## Plots in 3D

Functions of two variables, scalarfields $\left(F: \mathbb{R}^{2} \rightarrow \mathbb{R}\right)$ : Plot3D, ContourPlot, DensityPlot Special new features for 3D Graphics in Version 6.
3D Parametric curves and surfaces: ParametricPlot3D and its versions
Contoursurfaces of scalarfields $V: \mathbb{R}^{3} \rightarrow \mathbb{R}$
Examples
Plot functions in different coordinate systems
Visualization of moving points in 3D
$\square$ Graphics summary
Graphics structures in Version 6.
Graphics Structures in Version 5
Graphics conversions in Vesrsin 5
Examples
Simple volume rendering: plot and color the points of a volume Visualization of series expansions, spectrum animation
Basic applications in Mathematics

## Data manipulations

Nonlinear transformations on planar datasets
Graphical study of 2D experimental data: transformations, special plots

## Vectors, Matrices, Linear Algebra

Linear Algebra: vectors, matrices, transformations, eigenvalues, eigenvectors, etc.
$\square$ Calculus
Calculus summary
Applications amd examples in 1D Calculus
Tangent line and secant lines
Animation of the tangent lines
Animation of trigonometric series
Graphical interpretations of derivative: zooming

Investigation of functions
Taylor polynomials

## $\square$ Applications in 2D-3D Calculus

Calculus methods: partial derivatives, ..., maxima and minima
Tangent planes and normal vectors of surfaces
Tangent vectors and normal planes of curves
Moving points in 2D
Moving points in 3D
Maxima and minima: the numerical and visual point of view
Constrained extrema
$\square$ Curve Fitting
Curve fitting
$\square$ Complex numbers, complex functions
Summary and examples
$\square$ Differential equations I.
ODE summary
Modeling with 1D ODE's
2D equations: vector fields, solutions, trajectories
Trajectory animation and manipulation in 2D (using only built in functions)
Trajectory animation in 3D (Lorenz system)

## Advanced programming in Mathematica

## List programming I: Structure operations

Structure operations; Apply rules to lists

## List programming II

Rotating lists (RotateLeft, RotateRight)
Examples
The midpoint rule
Moving average of data and analogous problems
Simple image processing
ListConvolve, ListCorrelate
Recursion, iteration
Summary and basic examples
Examples: recursion vs. nesting
Factorial
Continued fractions
Simple numerical algorithms
Fixedpoints of mappings
Newton iterations
Picard iteration
Methods to approximate zeros of functions
Riemann sums

## Programming paradigms in Mathematica: a systematic treatment

Procedural programming
Functional programming
Rule- and pattern-based programming

## Nontrivial applications

$\square$ Addons' to list programming: some string manipulations
Examples
Morse
A coding-decoding system
$\square$ Graphical programming I: Advanced visualization
Transformations on graphical (Graphics and Graphics3D) objects
Transformations on Graphics objects (2D)
Transformations on Graphics3D objects (3D)
Replace anything by anything
Numerical data from graphical objects
Points of contour lines of planar scalar fields
Normal vectors to a graphically given surface
Advanced visualization problems (graphics from numerical data)
Volume rendering, slicing; coloring the discrete space by scalar fields
Volume rendering, slicing; coloring the continuous space by scalar fields
Visualizing parametric curves given in lists
Vector field at given contour lines in 2D
Vector field at given surfaces in 3D

## Graphical programming II: Advanced substitutions, Iterative forms

Simple iterative constructions
Iterate simple substitutions (generate trees)
Iterate the substitution of patterns
Simple selfsimilar objects
Some other examples
Iterations using the midpoint rule
Generate trees
Some more tree-like structures
Sierpinsky triangles, Koch curves
Sierpinsky attractor
Difference equations, finite differences
Difference equations: definition, solution
Cobweb diagram
Logistic mapping, bifurcation diagram
Solve and visualize planar difference equations
Partial difference equations (a procedural way of solution)
Application of list rotation to finite differences
Disretization of PDE's: more examples
Differential equations: advanced problems
Technical tools
General solver: ODESolve
Visualization of solutions given in lists: Euler's method
Qualitative methods
Stability: Liapunov method
The phasemap method
ODE's with Dirac delta
Poincare maps
Extra
Writing Modules and Packages
Package design, a general overview
Examples
Variableless mappings from expressions: EulerDSolveGeneral program usig variable names: ODESolveHandling options: Colored ParametricPlot
Additional topics. Notebook operations II
Options, option inspector, simple stylesheet development
Data export,import
Advanced style operations: automatic numbering, hyperlinks,...
Export, import: HTML, XML, MathML, TeX

## Exercises

Basic exercises
1D Calculus
Data handling, fitting
Linear Algebra
3D Claculus, Lines and Surfaces
1D differential equations
Oscillator equations
Planar differential equations
Programming exercises
Advanced programming exercises

