Content

Problems in Mathematics
&
Experiments with Mathematica

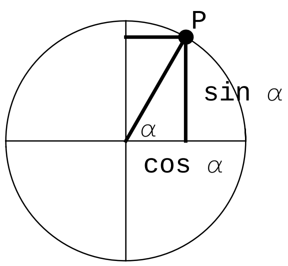
2. Elementary functions

2.4 Trigonometric functions

Theory

- **Definitions**

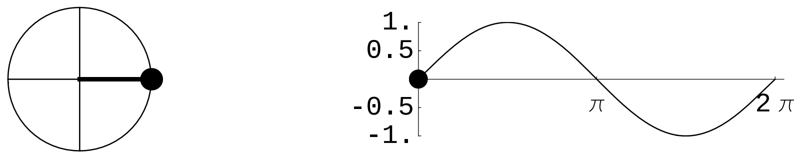
Let P be the intersection point of the unit circle with center at the origin and radius from the center at the angle α. By the definition, P=(cos α, sin α), that is the x-coordinate of P is cos α and the y-coordinate of P is sin α.



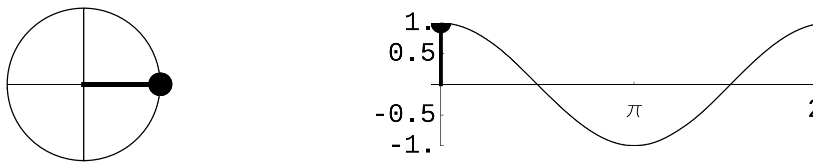
By definition, $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$, $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$, $\sec\alpha = \frac{1}{\cos\alpha}$ and $\csc\alpha = \frac{1}{\sin\alpha}$.

According to the usual notations for independent variables, we denote the argument of the trigonometric functions by x, where x is measured in radians.

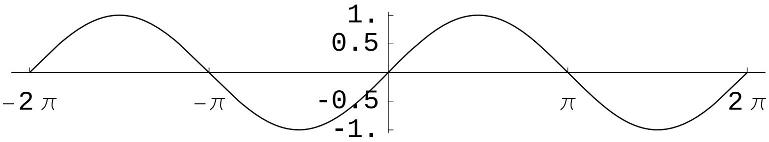
- *Animation: The definition of sin (x) (in the electronic version)*



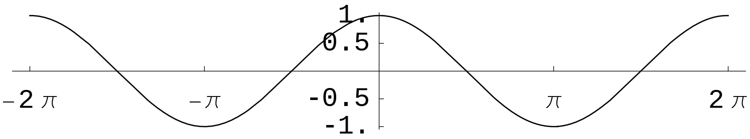
- *Animation: The definition of cos (x) (in the electronic version)*



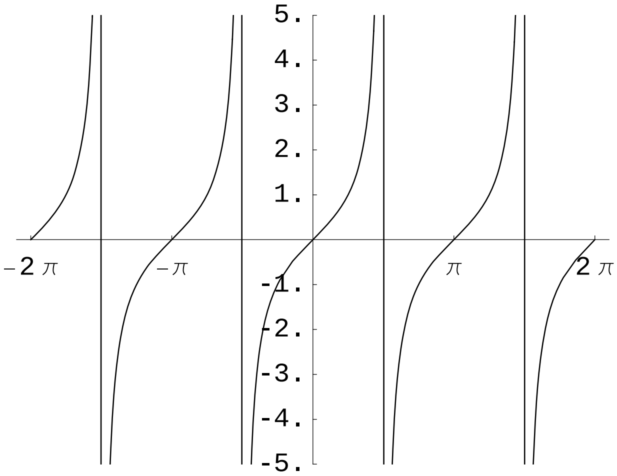
- **Graphs**
  - *sin x*



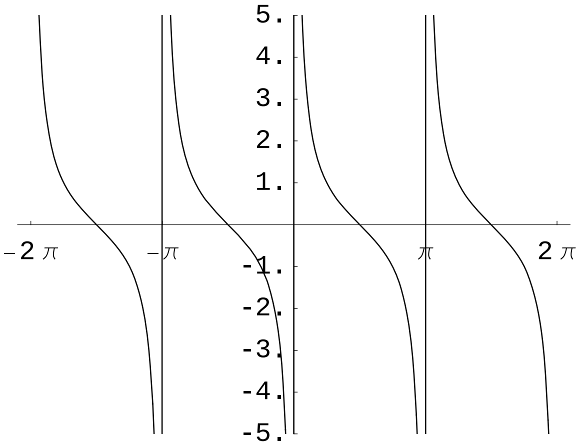
  - *cos x*



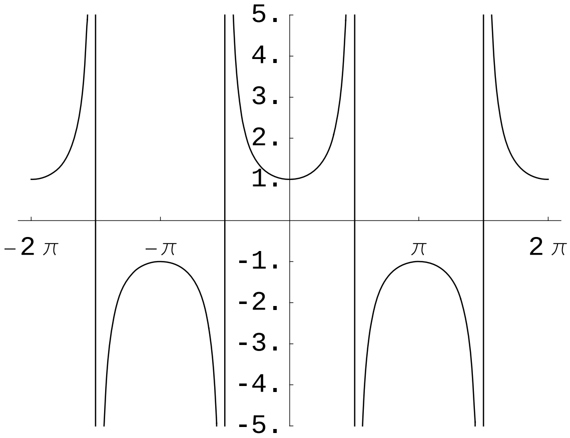
  - *tan x*



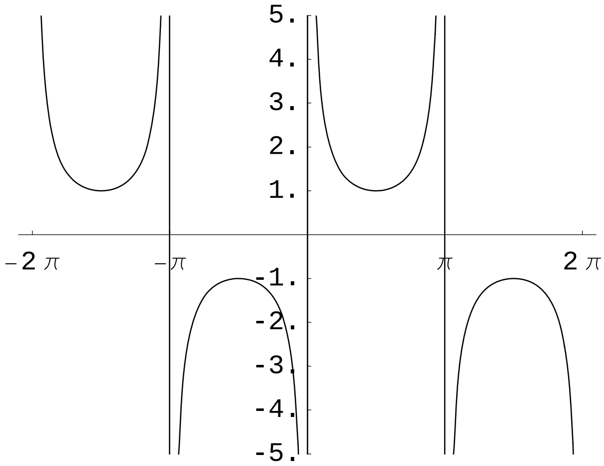
  - *cot x*



- *sec x*



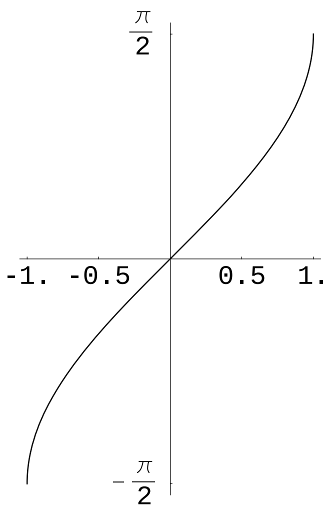
- *csc x*



• Inverses

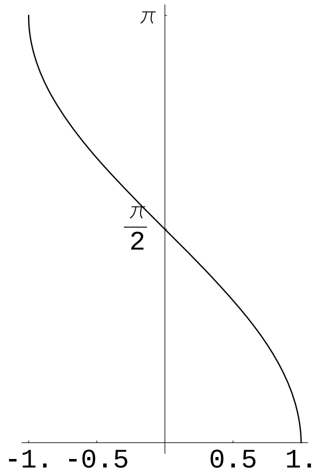
- *arcsin x*

The inverse of *sin(x)* on the interval [-$\frac{\pi}{2}$, $\frac{\pi}{2}$].



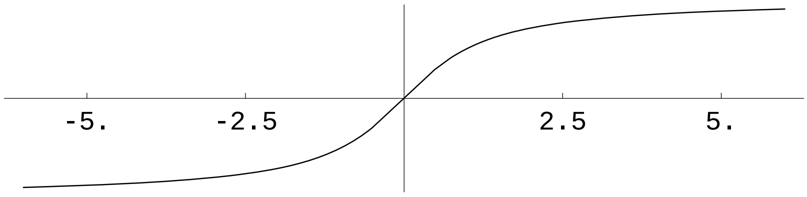
- *arccos x*

The inverse of *cos(x)* on the interval [0,π].



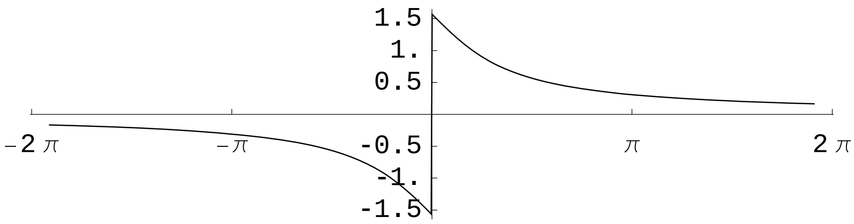
- *arctan x*

The inverse of *tan(x)* on the interval $(-\frac{\pi}{2},\frac{\pi}{2})$.



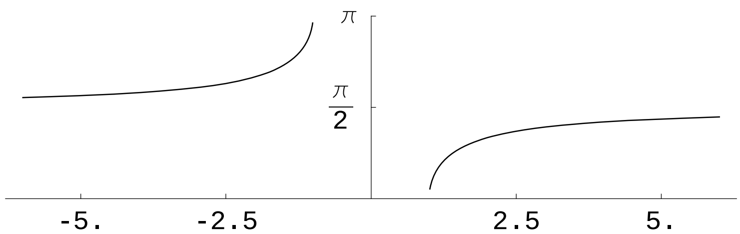
- *arccot x*

The inverse of *cot(x)* on the interval $(0,\pi)$.



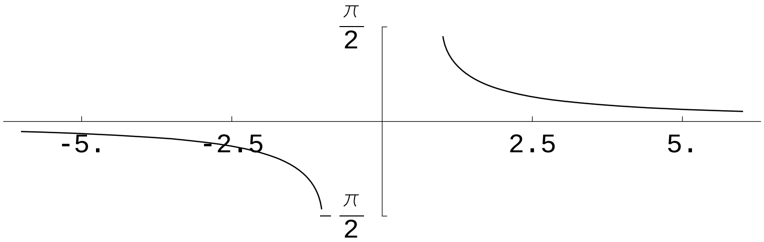
- *arcsec x*

The inverse of *sec(x)* on the interval $[0,\pi]$.



- *arccsc x*

The inverse of *csc(x)* on the interval $[-\frac{\pi}{2},\frac{\pi}{2}]$.



• Properties

Pythagorean Theorem: $\cos^2 x + \sin^2 x = 1$

Periodicity: $\cos(x + 2\pi) = \cos x$; sin (x+2π) =sin x;

$\tan(x + \pi) = \tan x$; cot (x+π) =cot x;

Symmetry: $\cos(-x) = \cos x$; sin (-x) =-sin x;

$\tan(-x) = -\tan x$; cot (-x) =-cot x;

Complementary angles:

$\cos x = \sin(\frac{\pi}{2} - x) = \sin(\frac{\pi}{2} + x)$;

$\cot x = \tan(\frac{\pi}{2} - x)$.

Addition theorems:

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Double angles:

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Mathematica statements

• *The trigonometric functions*

InputForm	StandardForm	TraditionalForm
Sin[x]	Sin[x]	sin (x)
Cos[x]	Cos[x]	cos (x)
Tan[x]	Tan[x]	tan (x)
Cot[x]	Cot[x]	cot (x)
Csc[x]	Csc[x]	csc (x)
Sec[x]	Sec[x]	sec (x)

• *The inverses of the trigonometric functions*

InputForm	StandardForm	TraditionalForm
ArcSin[x]	ArcSin[x]	$\sin^{-1}(x)$
ArcCos[x]	ArcCos[x]	$\cos^{-1}(x)$
ArcTan[x]	ArcTan[x]	$\tan^{-1}(x)$
ArcCot[x]	ArcCot[x]	$\cot^{-1}(x)$
ArcCsc[x]	ArcCsc[x]	$\csc^{-1}(x)$
ArcSec[x]	ArcSec[x]	$\sec^{-1}(x)$

Plot statements

• *Plot bounded trigonometric functions*

```
Needs["Graphics`Graphics`"];
```

```
Plot[Sin[x], {x, -2 π, 2 π}, AspectRatio → Automatic,
Ticks → {PiScale[#1, #2, 4] &, UnitScale[#1, #2, 0.5, 10] &}];
```

- *Plot unbounded trigonometric functions*

```
Plot[Tan[x], {x, -2 π, 2 π},
  AspectRatio → Automatic, PlotRange → {-5, 5},
Ticks → {PiScale[#1, #2, 4] &, UnitScale[#1, #2, 0.5, 10] &}];
```

- *Plot the inverses*

```
Plot[ArcCos[x], {x, -1, 1}, AspectRatio → Automatic,
Ticks → {UnitScale[#1, #2, 0.5, 10] &, PiScale[#1, #2, 4] &}];
```