Content

Problems in Mathematics
&
Experiments with Mathematica

7. 1D Ordinary differential equations

7.2 *Mathematica* tools to solve differential equations

Formal solution of a 1D equation

- *Finding the general solution*

```
DSolve[x'[t] == f[t, x[t]], x[t], t]
```

The general solution is provided in a form of a substitution rule. The right-hand side can contain parameters. The solution contains an integration parameter *C[1]* that has to be eliminated when solving an initial value problem.

- *Finding the solution of an initial value problem*

```
DSolve[x'[t] == f[t, x[t], x[t0] == x0}, x[t], t]
```

See more details about DSolve in the *Mathematica* help.

SOLVED PROBLEM 7.2.1

Solve the initial value problem

$$\mathbf{eqn} = x'(t) == x(t)\,(2 - x(t));$$
$$t_0 = 0;\ \ x_0 = 1;$$

○ SOLUTION 1. Follow the manual way.

- *The general solution*

```
gensol = DSolve[eqn, x[t], t]
```

$$\left\{\left\{x[t] \to \frac{2\,E^{2\,t}}{E^{2\,t} - E^{2\,C[1]}}\right\}\right\}$$

- *Find the value for C[1] such that x[t0]=x0*

```
Cval = Solve[(x[t] /. gensol /. t -> t₀) == {x₀}, C[1]]
```

$$\left\{\left\{C[1] \to -\frac{I\,\pi}{2}\right\}, \left\{C[1] \to \frac{I\,\pi}{2}\right\}\right\}$$

The obtained C[1] solutions are complex. It is correct, substituting into the general solution, we obtain the desired real solution.

- *The solution of the initial value problem*

```
sol = x[t] /. gensol /. Cval
```

$$\left\{\left\{\frac{2\,E^{2\,t}}{1+E^{2\,t}}\right\},\ \left\{\frac{2\,E^{2\,t}}{1+E^{2\,t}}\right\}\right\}$$

Or, in a function form

```
ivsol[t_] = sol[[1, 1]]
```

$$\frac{2\,E^{2\,t}}{1+E^{2\,t}}$$

- *All these steps in one command*

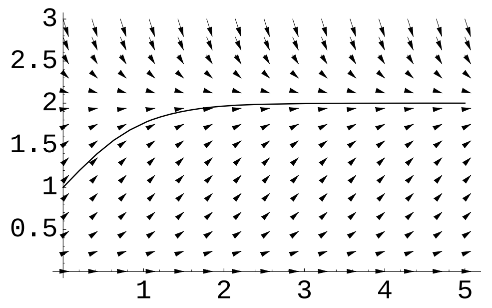
```
ivsol =  x[t] /. (gensol = DSolve[eqn, x[t], t] ) /.
(Crul = Solve[(x[t] /. gensol /. t -> t0) == {x0}, C[1]])[[1]]
```

$$\left\{\frac{2\,E^{2\,t}}{1+E^{2\,t}}\right\}$$

- *Plot the solution and the direction field*

```
Needs["Graphics`PlotField`"]
```

```
plsol = Plot[ivsol[t], {t, 0, 5}, DisplayFunction -> Identity];
plfield = PlotVectorField[{1, x (2 - x)},
   {t, 0, 5}, {x, 0, 3}, DisplayFunction -> Identity];
Show[plsol, plfield, DisplayFunction -> $DisplayFunction];
```



○ SOLUTION 2. Use the features of *Mathematica*.

```
Clear[x];
ivpsol = x[t] /. DSolve[{eqn, x[t0] == x0}, x[t], t]
```

$$\left\{\frac{2\,E^{2\,t}}{1+E^{2\,t}}\right\}$$

Numerical solution of a 1D equation

```
NDSolve[x'[t] == f[t, x[t], x[t0] == x0, x[t], {t, t0, T}]
```

The solution of the initial value problem is given on the interval *[t0,T]*. The solution is an *InterpolatingFunction* object.
See more details on `NDSolve` and `InterpolatingFunction` object in the *Mathematica* help.

SOLVED PROBLEM 7.2.2

Solve the above initial value problem numerically on the interval *[t0,T]*.

Clear(x)**;**
eqn $= x'(t) == x(t)\,(2 - x(t))$**;**
$t_0 = 0$; $x_0 = 1$; $T = 5$;

○ SOLUTION

- *Solve the equation*

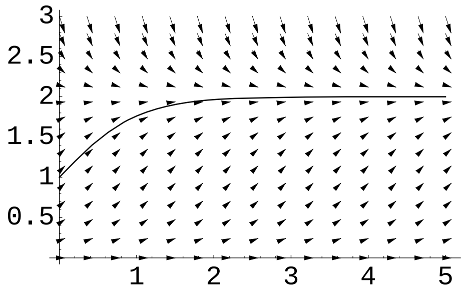
```
nsol[t_] = x[t] /. NDSolve[{eqn, x[t0] == x0}, x[t], {t, t0, T}]
```

```
{InterpolatingFunction[{{0., 5.}}, <>][t]}
```

- *Plot the solution and the direction field.*

```
Needs["Graphics`PlotField`"]
```

```
plsol = Plot[nsol[t], {t, 0, 5}, DisplayFunction -> Identity];
plfield = PlotVectorField[{1, x (2 - x)},
   {t, 0, 5}, {x, 0, 3}, DisplayFunction -> Identity];
Show[plsol, plfield, DisplayFunction -> $DisplayFunction];
```



○