Content

Problems in Mathematics & Experiments with Mathematica

2. Elementary functions

2.10 Other transformations: $|f(x)|$, $\frac{1}{f(x)}$, $f\left(\frac{1}{x}\right)$

Theory

Formula	Transformation
$\lvert f(x) \rvert$	The negative part ($y < 0$) is mirrored to the x – axis.
$\frac{1}{f(x)}$	Applies the reciprocal for the value of the function.
$f\left(\frac{1}{x}\right)$	Exchanges the definition on $(0, 1)$ and $[1, \infty)$.

Simple exercises, problems

Mathematica initialization

```
<< "Graphics`Graphics`"
```

Solved Problem 2.10.1 The graph of |f(x)|

Plot the graph of the function

$$f(x\_) := |(x-1)(x-2)|;$$

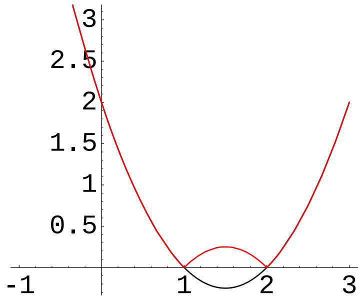
○ Solution

The negative part of the graph of the function

$$g(x\_) := (x-1)(x-2);$$

is mirrored to the x-axis. Plot the original and the transformed functions:

```
x0 = -1; x1 = 3; Plot[{g[x], Abs[g[x]]}, {x, x0, x1},
AspectRatio → Automatic,
PlotStyle → {{Hue[1, 1, 0]}, {Hue[1]}}];
```



PROBLEM 2.10.2

Plot the absolute value of the following functions.

(1) $g(x\_) := \log_2(x);$

(2) $g(x\_) := (x+2)(x-1)(x-3);$

(3) $g(x\_) := \sin(x);$

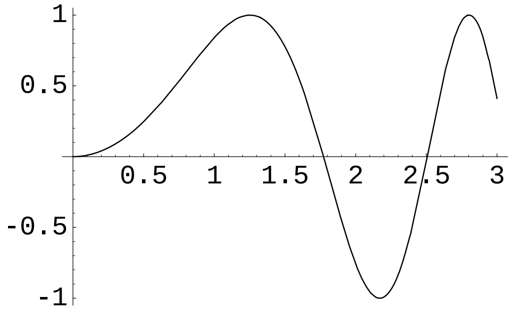
(4) $g(x\_) := \tan(x);$

(5) $g(x\_) := \dfrac{x-1}{x-2};$

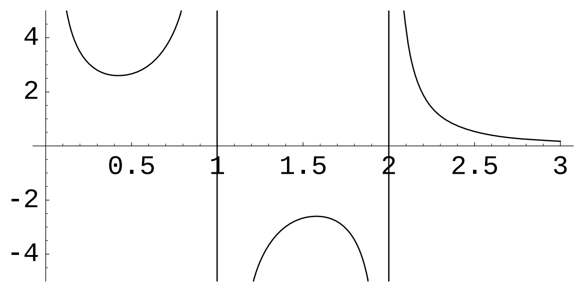
PROBLEM 2.10.3

Plot the absolute value of the following functions.

(1)



(2)



SOLVED PROBLEM 2.10.4 The transformation $\frac{1}{f(x)}$

Plot the graph of the function

$$f(x\_) := \frac{1}{(x-1)(x-2)^2(x-3)}$$

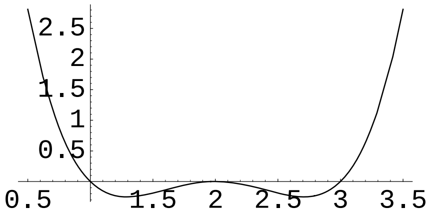
○ SOLUTION

Take the reciprocal of the function to obtain the graph of *f(x)*.

$$g(x\_) := (x-1)(x-2)^2(x-3)$$

Plot the original function:

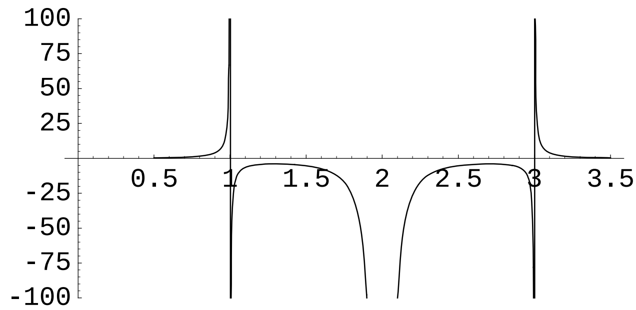
```
x0 = 0.5; x1 = 3.5; Plot[{g[x]}, {x, x0, x1},
    AspectRatio → 1/2, PlotRange → All,
    PlotStyle → {Hue[1, 1, 0]}];
```



Be careful! The zeros of *g(x)* are critical. The function $\frac{1}{g(x)}$ is not defined, and the graph is broken at these places. Observe that *g(x)* changes the sign from + to - at 1, from - to + at 3, and it does not change the sign at 2.

Plot the transformed function:

```
x0 = 0.5; x1 = 3.5;
Plot[{1/g[x]}, {x, x0, x1},
    AspectRatio → 1/2, PlotRange -> {-100, 100},
    PlotStyle → {Hue[1, 1, 0]}];
```



○

PROBLEM 2.10.5

Plot the graph of the reciprocal of the following functions:

(1) $f(x\_) := x^3 - 1;\quad g(x\_) := \log_{\frac{1}{2}}(x);$

(2) $f(x\_) := 2^{(x+1)};\quad g(x\_) := (1 - x^2)(x-2);$

(3) $f(x\_) := x^2\cos(x);\quad g(x\_) := \sin(x);$

SOLVED PROBLEM 2.10.6 Transformation $f\left(\frac{1}{x}\right)$, inversion

Plot the graph of the function

$$f(x\_) := \sin\left(\frac{1}{x}\right);$$

○ SOLUTION

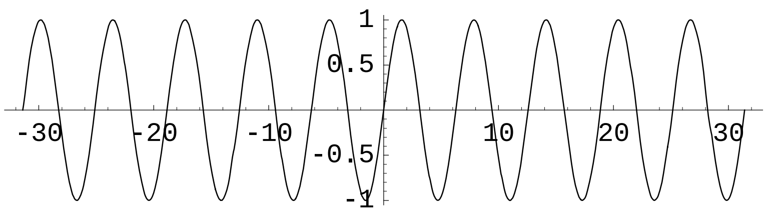
To obtain the graph of *f(x)*, the definition of

```
g[x_] := Sin[x]
```

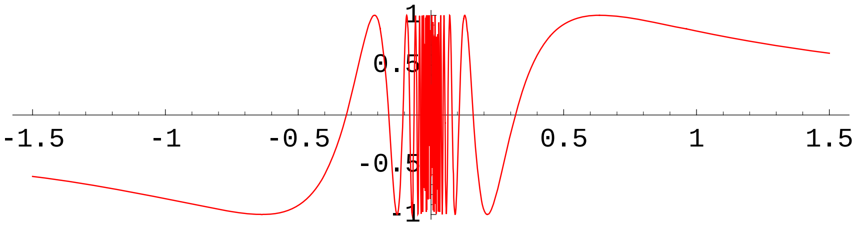
on *[1,∞)* and *(0,1]* are exchanged. The graph of *g(x)* close to zero will be the graph of *f(x)* close to infinity and vica versa.

Plot the original and the transformed functions separately:

```
x0 = -10 π; x1 = 10 π;
Plot[{g[x]}, {x, x0, x1},
    AspectRatio → 1 / 4, PlotRange -> All,
    PlotStyle → {Hue[1, 1, 0]}];
```

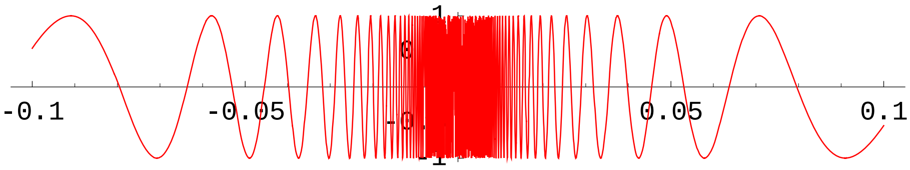



```
x0 = -1.5; x1 = 1.5;
Plot[g[1 / x], {x, x0, x1},
    AspectRatio → 1 / 4, PlotRange -> All, PlotPoints -> 100,
    PlotStyle → {Hue[1]}];
```



The graph on *[1,1.5]* is clear. This corresponds to the interval *[2/3,1]* of the graph of *g(x)*. But the interval *[-1,1]* is a little bit messy. Really, since the places nπ *(n=0,±1,±2,±3,...)* are the zeros of *g(x)*, the places *(n=±1,±1/2,±1/3,...)* are the zeros of *g(1/x)*. Since *g(x)* has infinitely many waves on *[1,∞)*, and *g(1/x)* has infinitely many waves on *(0,1]*. Zoom in a neighborhood of the origin:

```
x0 = -0.1; x1 = 0.1;
Plot[g[1/x], {x, x0, x1},
    AspectRatio → 1/6, PlotRange -> All, PlotPoints -> 200,
    PlotStyle → {Hue[1]}];
```



Enlarge the image to find a better resolution: select the graph and the drag one of the corners of the bounding box. An even better resolution can be obtained by increasing the PlotPoints.

PROBLEM 2.10.7

Plot the graph of $f\left(\frac{1}{x}\right)$ and $g\left(\frac{1}{x}\right)$ for the following functions.

(1) $f(\text{x\_}) := \tan(x);\ \ g(\text{x\_}) := \cos(3\,x);$

(2) $f(\text{x\_}) := \cos\left(\frac{1}{x}\right);\ \ g(\text{x\_}) := x^3 - 1;$

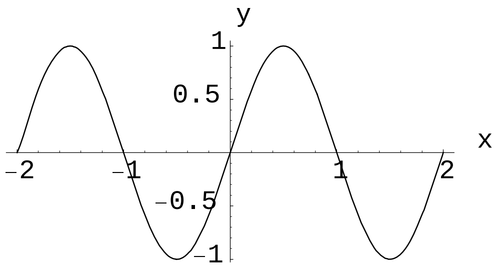
(3) $f(\text{x\_}) := \sqrt{x} + 3;\ \ g(\text{x\_}) := \sqrt[3]{x-1};$

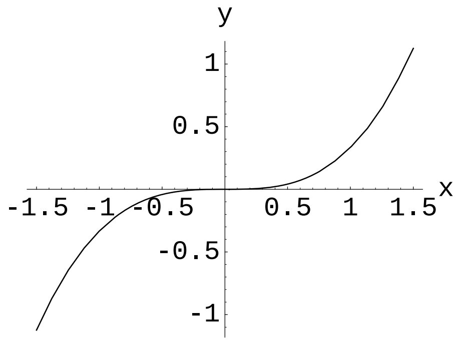
(4) $f(\text{x\_}) := \log_3(x);\ \ g(\text{x\_}) := \log_3(-x);$

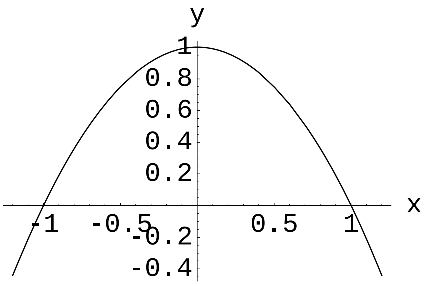
(5) $f(\text{x\_}) := 2^{-x};\ \ g(\text{x\_}) := 2^x;$

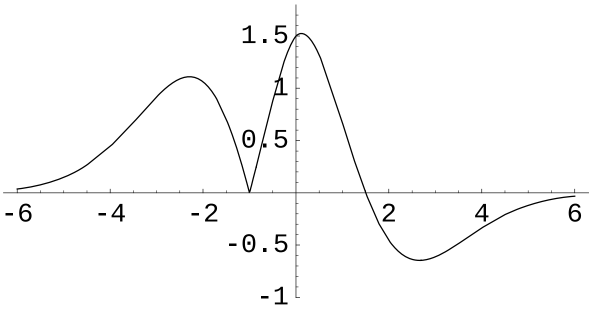
PROBLEM 2.10.8

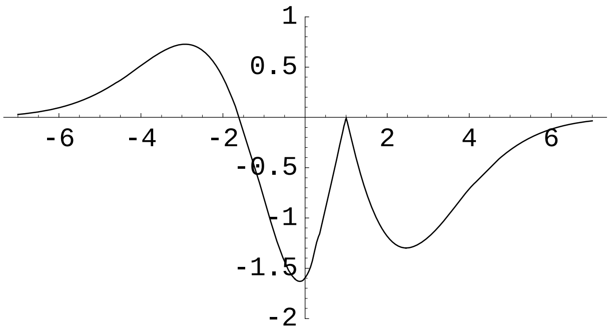
Plot the graph of $f\left(\frac{1}{x}\right)$, $f(|x|)$ and $f(x)^2$ for the following functions.

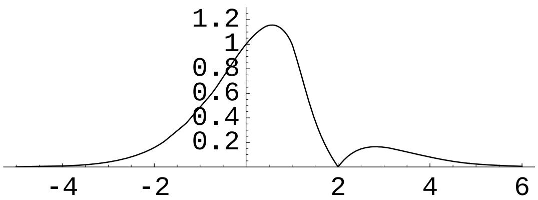





y
1
0.8
0.6
0.4
0.2
x
-1
-0.5
0.5
1
-0.2
-0.4


1.5
1
0.5
-6
-4
-2
2
4
6
-0.5
-1


1
0.5
-6
-4
-2
2
4
6
-0.5
-1
-1.5
-2


1.2
1
0.8
0.6
0.4
0.2
-4
-2
2
4
6