Problems in Mathematics & Experiments with Mathematica

6. Definite integral

6.2 Formal integration: Newton-Leibniz formula

Theory

THEOREM 6.2.1 Recall the Fundamental Theorem

Let *f(x)* be integrable on the interval *[a,b]*. If *f(x)* is continuous at $x_0 \in (a, b)$, then the area function $\int_a^x f(x)\, dx$ is differentiable at x_0, and

$$\boxed{\frac{d}{dx}\left(\int_a^x f(x)\, dx\right)\bigg|_{x=x_0} = f(x_0)}$$

Consequently, the area function is an antiderivative of *f(x)*.

See the properties of the definite integral in more details in the section Definition, geometrical meaning, properties.

THEOREM 6.2.2 Newton-Leibniz formula

The Fundamental Theorem implies the basic tool to calculate definite integrals. Let *f(x)* be continuous on the interval [a,b], and let *F(x)* be an antiderivative of *f(x)*. Then

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

- *Integration steps*

  Find an antiderivative, piecewise if necessary.
  Apply the Newton-Leibniz formula.

For finding an antiderivative, see the chapter Integration techniques.

Mathematica summary

Inputform	Traditional Form	Meaning
Integrate[f[x], {x, a, b}]	$\int_a^b f(x)\,dx$	Definite integral of $f(x)$ on $[a, b]$

Exercises and problems

PROBLEM 6.2.1

(1) $\int_{-1}^{2}(x^2 - x)\,dx$

(2) $\int_{0}^{\pi}\sin(x)\,dx$

(3) $\int_{1}^{10}\log(x)\,dx$

(4) $\int_{0}^{5}(e^{-2x} + 1)\,dx$

(5) $\int_{0}^{\pi}x^2\sin(x)\,dx$

(6) $\int_{0}^{1}e^x(x^2 - x)\,dx$

(7) $\int_{0}^{\frac{\pi}{2}}x\tan(x^2)\,dx$

(8) $\int_{-2}^{2}x\,e^{-x^2}\,dx$

(9) $\int_{e}^{e^2}\frac{1}{x\log(x)}\,dx$

PROBLEM 6.2.2

Calculate the following integrals. Verify your calculations by performing the *Mathematica* commands.

(1) $\int_{-1}^{2}|x^2 - 2x|\,dx$

(2) $\int_{0.5}^{2}|x\log(x)|\,dx$

(3) $\int_0^{\pi} |\sin(6\,x)| \, dx$

(4) $\int_0^{2\pi} (|x-1| - |x\sin(x)|) \, dx$