Content

Problems in Mathematics
&
Experiments with Mathematica

Elementary functions

Exercises on elementary functions

Problems

Domain and graph

Give the domain and plot the graph of the following functions.

$$\cot(x); \quad \sqrt{x+2}; \quad \frac{1}{x^2}; \quad \log(-x)-2; \quad 1-3^{-x};$$

$$\log(x); \quad \frac{1}{x-1}; \quad \sqrt[3]{|x|}; \quad (x-2)^3-3; \quad 1-3^x;$$

$$2^x; \quad -\sqrt{x-3}; \quad \frac{1}{|x|}; \quad 1-\log(x); \quad 2-4^{-x};$$

$$\tan(x); \quad \frac{1}{x}; \quad \sqrt{x}-1; \quad \log(|-x|); \quad 2-2^x;$$

$$1+\frac{1}{(2-x)^2}; \quad \tan(2\pi x-\pi); \quad \log_2(1-x)-2; \quad \cot(\pi x+1);$$

$$1+\frac{1}{(2-x)^2}; \quad \tan(2\pi x-\pi); \quad \log_2(1-x)-2; \quad \cot(\pi x+1);$$

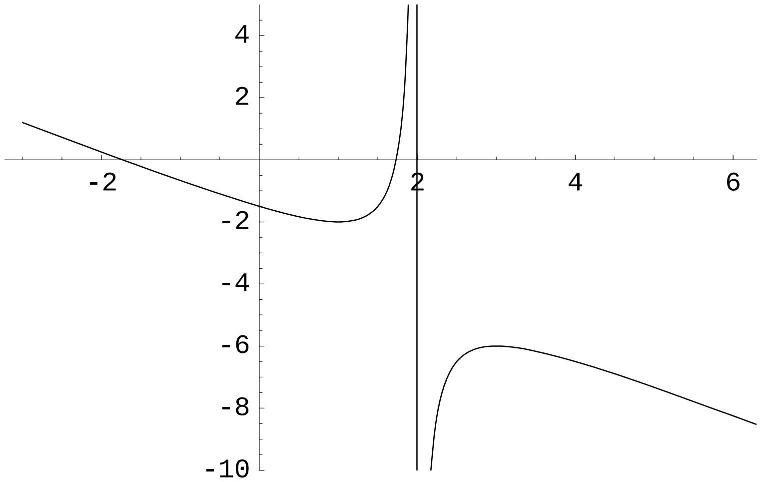
What is the domain of the following functions?

$$f(x\_) := \frac{3-x^2}{x-2}$$

- SOLUTION

  $D=R\backslash\{2\}$

```
Plot[f[x], {x, -3, 9}, PlotRange → {-10, 5}];
```



(2) $f(\mathrm{x\_}) := \dfrac{2}{x-1}$

(3) $f(\mathrm{x\_}) := \sqrt{x+2}$

(4) $f(\mathrm{x\_}) := \sqrt{-x^2 - 2\,x + 5}$

(5) $f(\mathrm{x\_}) := \dfrac{1}{x^2 - 7}$

(6) $f(\mathrm{x\_}) := \sqrt{\dfrac{1}{x^2 - 7}}$

(7) $f(\mathrm{x\_}) := \log(x+3)$

(8) $f(\mathrm{x\_}) := \log\left(\dfrac{x+5}{x-5}\right)$

(9) $f(\mathrm{x\_}) := \log(x^2 + 2\,x - 7)$

(10) $f(\mathrm{x\_}) := \sin^{\frac{1}{2}}(x)$

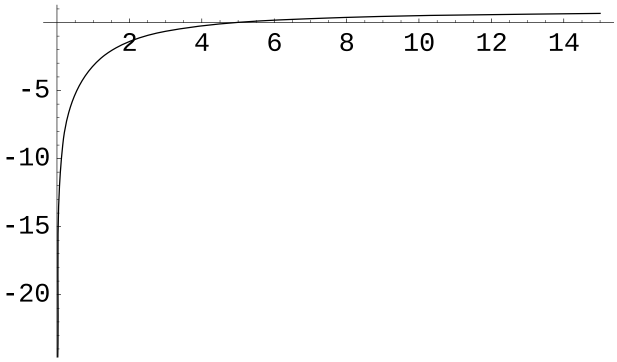
PROBLEM 2.6.3

Which function has the domain $D = \mathbb{R} \backslash \{0\}$?

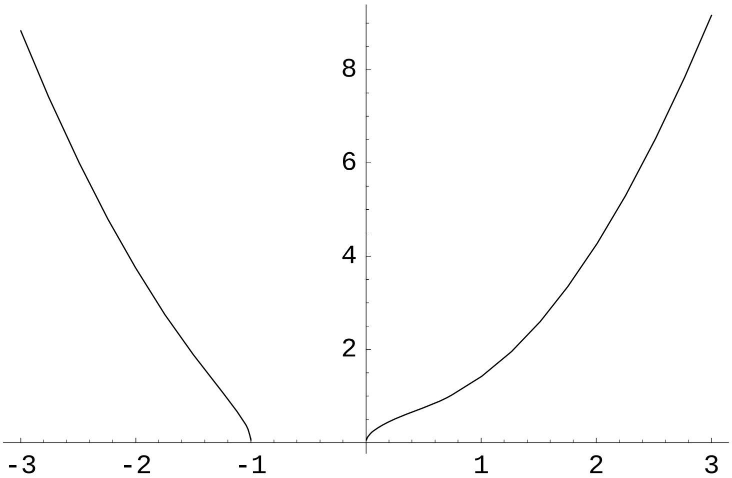
(1) $f(\mathrm{x\_}) := \dfrac{x-5}{\sqrt[3]{x^3 + x}}$; $g(\mathrm{x\_}) := \sqrt{x^4 + x}$; $h(\mathrm{x\_}) := \dfrac{x+2}{x\,\sqrt{x+2}}$;

○ SOLUTION

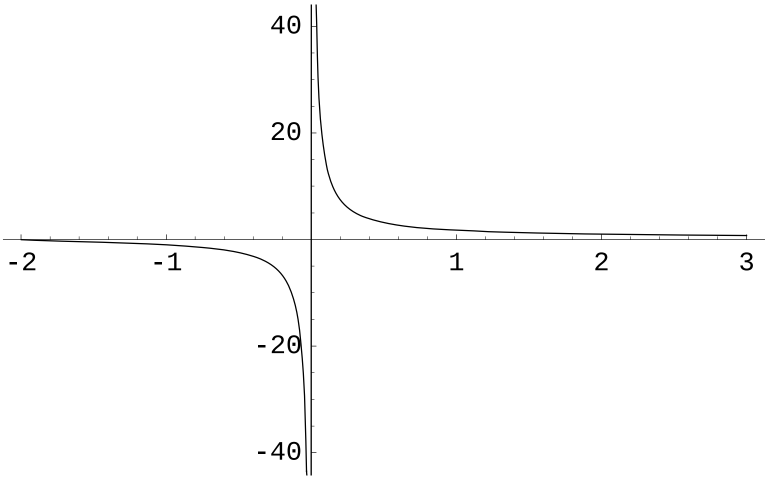
```
Plot[f[x], {x, 0, 15}];
```




```
Plot[{g[x]}, {x, -3, 3}];
```




```
Plot[{h[x]}, {x, -3, 3}];
```



Finding zeros

PROBLEM 2.6.4 Find the roots of the following functions formally

Find the roots of the following functions formally. Use Mathematica command **Solve** to check your result.

(1) $f(x\_) := x^2 - 2x;$

- SOLUTION

```
Solve[f[x] == 0, x]
```

```
{{x → 0}, {x → 2}}
```

(2) $f(x\_) := x^2 - 3x + 1;$

(3) $f(x\_) := -2x^2 - 6x + 1;$

(4) $f(x\_) := x^3 + 3x^2;$

(5) $f(x\_) := x^4 - 4x^2 + 1;$

(6) $f(x\_) := \sin(x^2);$

(7) $f(x\_) := e^{2x} - 9e^x + 2;$

(8) $f(x\_) := \log(\log(\log(x)));$

(9) $f(x\_) := \sin(\log(x));$

Finding zeros approximately

For polynomials of higher orders and for transcendent functions, there is no general rule to find the zero positions, moreover, it is impossible in most cases. We need numerical approximations to find approximative roots. The reader will find such methods among the application of the derivative in the chapter Approximation of zeros.