Problems in Mathematics & Experiments with Mathematica

2. Elementary functions

2.7 Inverse function

Theory

- **Recall the Definition**

If the function f is one-to-one, there exists a function g for which $g(f(x))=x$. This function is called the inverse function to $f(x)$ and denoted by $f^{-1}(x)$ or $\overline{f}(x)$.

Since the graph of $f(x)$ is the set of points $\{(x,f(x))\}$, the graph of the inverse function is the set of points $\{(f(x),x)\}$. Graphically, the graphs are mirror images of each other for the mirror line $y=x$.
It is obvious that the domain of $f^{-1}(x)$ is the range *of* $f(x)$, and the range of $f^{-1}(x)$ is the domain of $f(x)$.

To find the formula for the inverse
- solve the equation $y=f(x)$ for x.
- change the variables $x \longleftrightarrow y$

The obtained function is the inverse function.

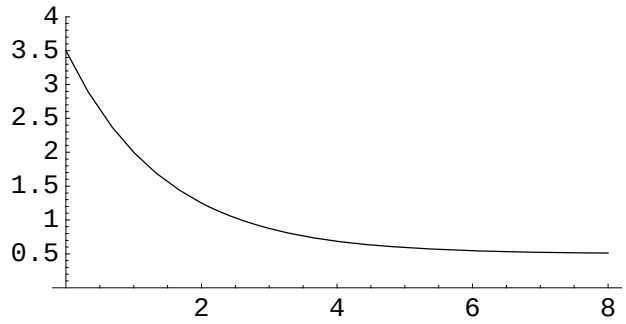
Mathematica statements

`Solve[eqn, var]`	Solves an equation
`Plot[ ...]`	Plots a graph of a function
`ParametricPlot[ ...]`	Plots a parametric curve

Problems and exercises

SOLVED PROBLEM 2.7.1 Inverse values graphically

Knowing that the graph of $f(x)$ is



find that place, where *f(x)=1*.

- *Remark*

An interpretation of this problem is important. The amount of a drug in the body at time *t* is described by the given curve. Find the time elapsed after the administration of the tablets till the moment, when the drug reaches a critical level.

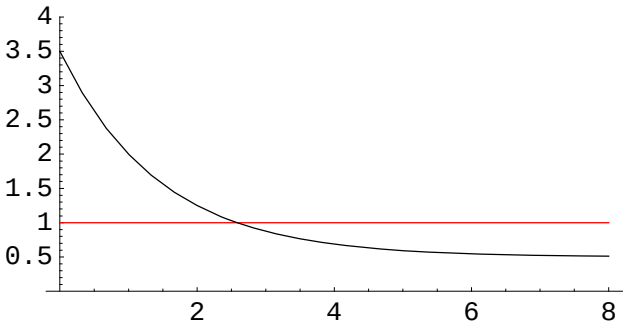
○ SOLUTION

Note that the graph was generated by the following command.

```
Plot[3 2^-x + 0.5, {x, 0, 8},
  PlotRange → {0, 4}, AspectRatio → Automatic];
```

To solve the problem, we have to draw the line *y=1*, and read the *x*-coordinate of the intersection point with the graph:

```
Plot[{1, 3 * 2^-x + 0.5}, {x, 0, 8}, PlotRange → {0, 4},
  AspectRatio → Automatic, PlotStyle → {Hue[1], Hue[1, 1, 0]}];
```



An approximate solution is *x=2.2*. Although we know the formula in this case, we do not solve the equation *f(x)=1*. This method can be used for experimentally given functions.

SOLVED PROBLEM 2.7.2 Graph and formula

Find the graph and the formula of the inverse of the following function:

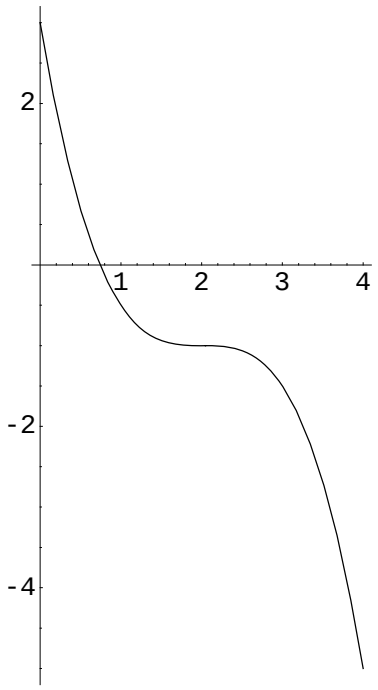
$$f(x\_) := -\frac{1}{2}(x-2)^3 - 1$$

○ SOLUTION

- *Plotting the graphs*

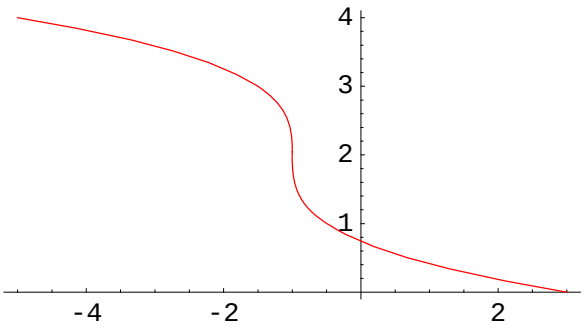
The graph of *f(x)* is

```
x0 = 0; x1 = 4;
p1 = Plot[f[x], {x, x0, x1}, AspectRatio -> Automatic,
     PlotRange -> All];
```



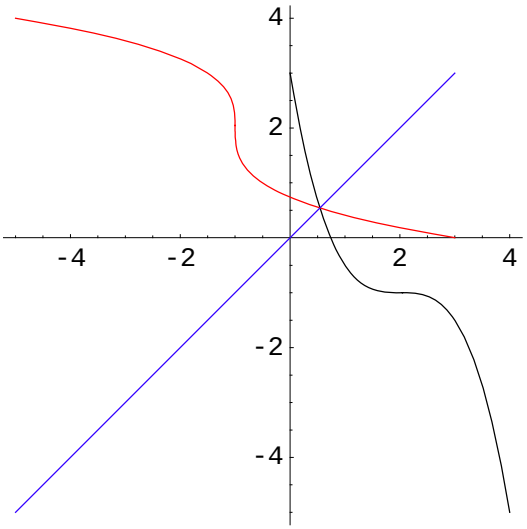
The graph of the inverse is as follows:

```
p2 = ParametricPlot[{f[x], x}, {x, x0, x1},
    AspectRatio -> Automatic, PlotRange -> All,
     PlotStyle -> {Hue[1]}];
```



Plot them together with the mirror line *y=x:*

```
mline = Plot[x, {x, f[x0], f[x1]}, PlotStyle -> {Hue[0.7]},
    DisplayFunction -> Identity];
Show[p1, p2, mline];
```



- *Formula for the inverse*

```
roots = y /. Solve[f[y] == x, y]
```

$$\left\{2+2^{1/3}\ (-1-x)^{1/3},\ 2-\frac{(1-I\sqrt{3})\ (-1-x)^{1/3}}{2^{2/3}},\ 2-\frac{(1+I\sqrt{3})\ (-1-x)^{1/3}}{2^{2/3}}\right\}$$

The inverse of *f(x)* is the first real root.

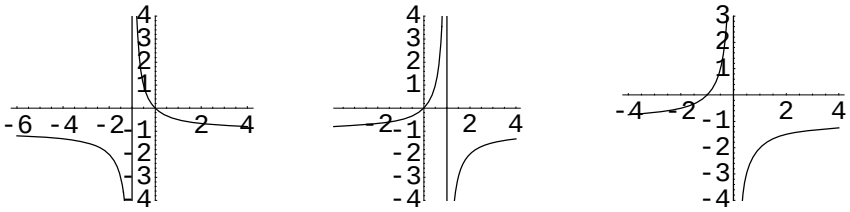
```
finv[x_] = roots[[1]]
```

$$2+2^{1/3}\ (-1-x)^{1/3}$$

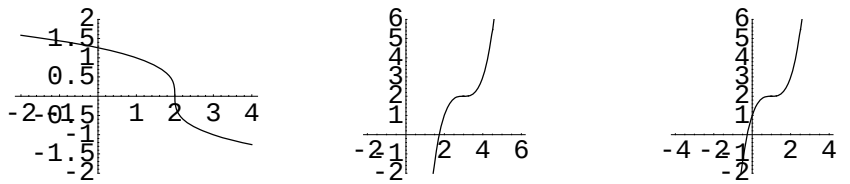
PROBLEM 2.7.3

Choose the graph of the inverse of the function *f(x)* from the given graphs.

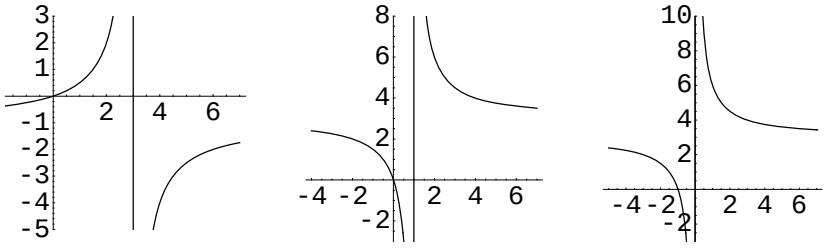
(1) $f(x\_) := \dfrac{x}{x+1};$



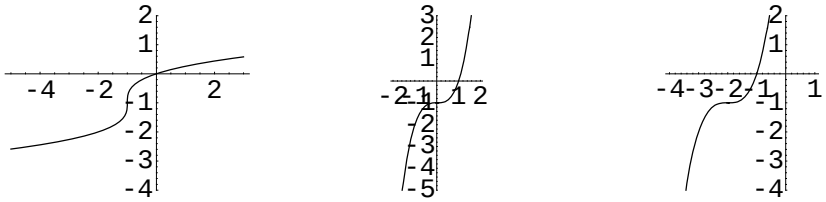
(2) $f(x\_) := \sqrt[3]{x-2}+1;$



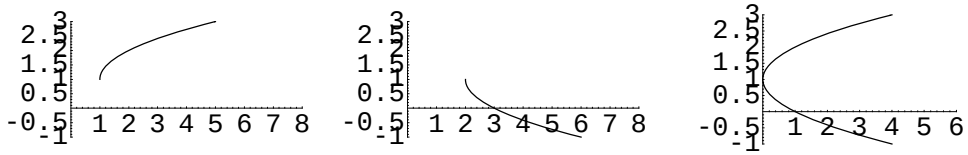
(3) $f(x\_) := \frac{x}{x-3};$



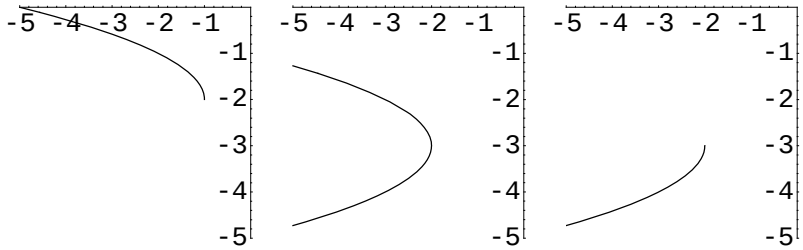
(4) $f(x\_) := \sqrt[3]{x+1} - 2;$



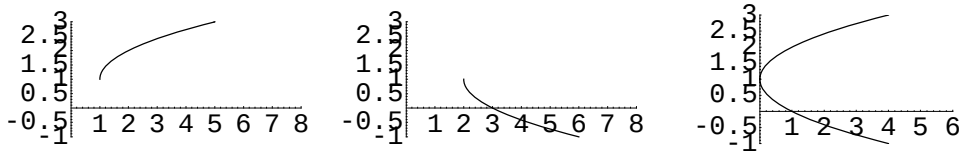
(5) $f(x\_) := (x-1)^2 + 2;$



(6) $f(x\_) := -(x+3)^2 - 2;$



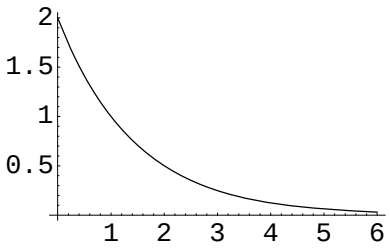
(7) $f(x\_) := (x-1)^2 + 2;$



PROBLEM 2.7.4

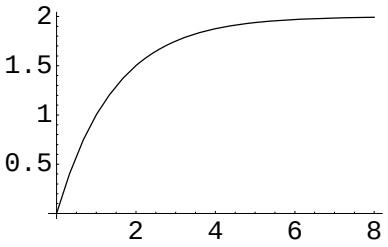
The function $f(x)$ is given by graph. Where does $\overline{f}(x)$ take y_1 and y_2?

(1)



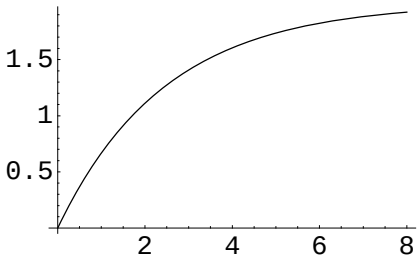
y_1 = 1; y_2 = 1.5;

(2)



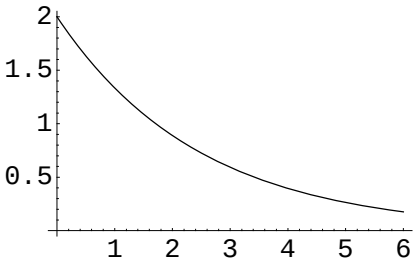
y_1 = 0.5; y_2 = 1.5;

(3)



y_1 = 0.5; y_2 = 1;

(4)



y_1 = 0.5; y_2 = 1;