[Content]

Problems in Mathematics
&
Experiments with Mathematica

2. Elementary functions

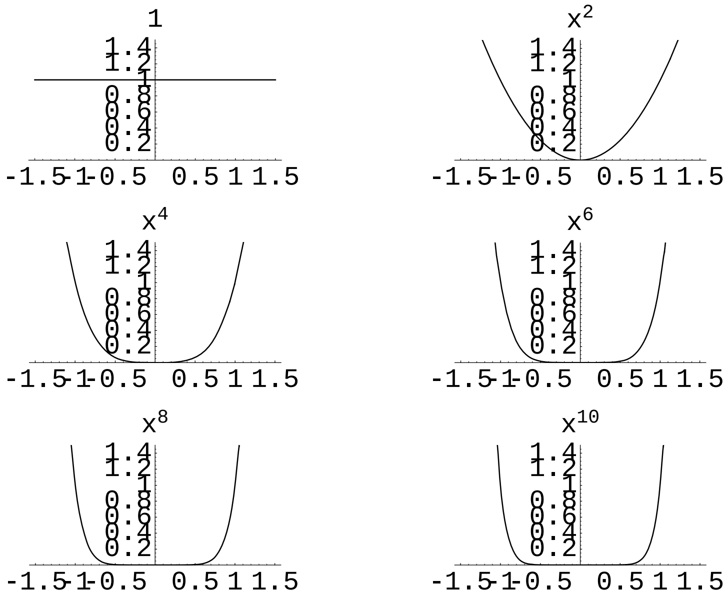
2.3 Powers

Powers with nonnegative integer exponent

- **Definition**

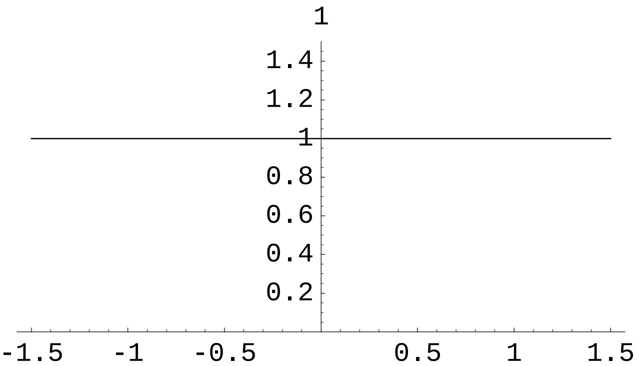
$x^n = x\,x\ \ldots\ x$	multiply x by itself n-times
$x^0 = 1$	by definition

- **Graphs for even exponents *(n=2k)***

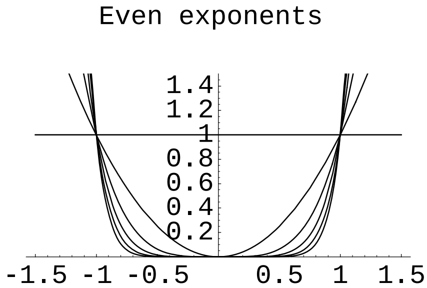


- *Animation (electronic version only)*

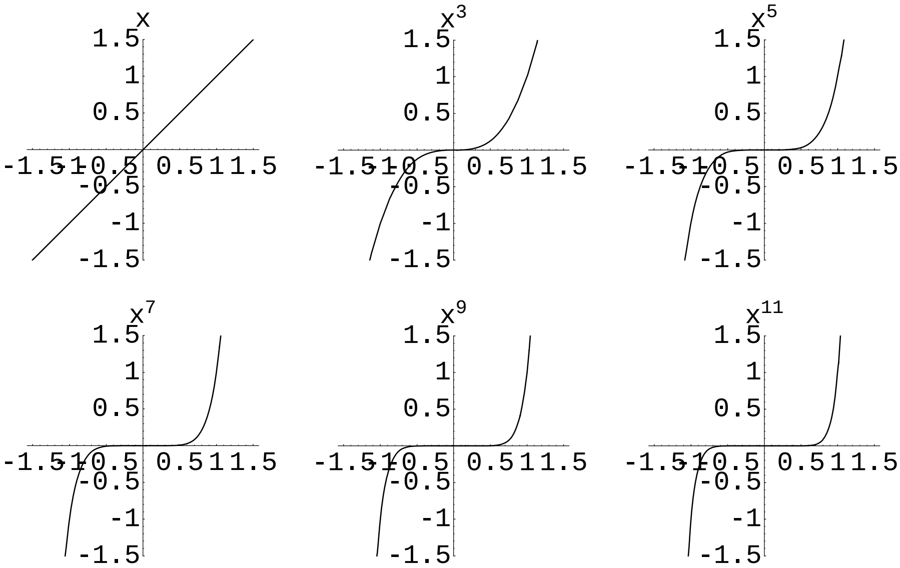
To animate the graphs, double-click on the first plot.



- *The graphs together*

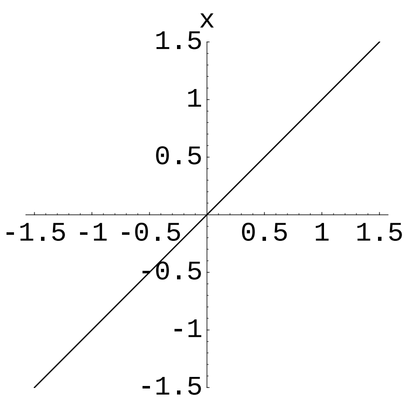


- **Graphs for odd exponents *(n=2k+1)***

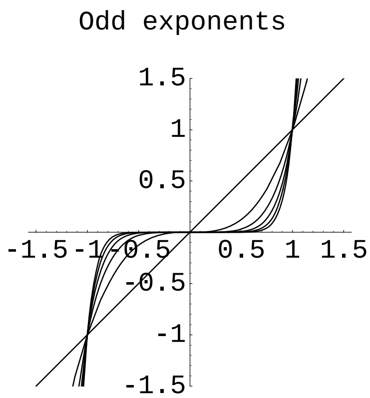


- *Animation (electronic version only)*

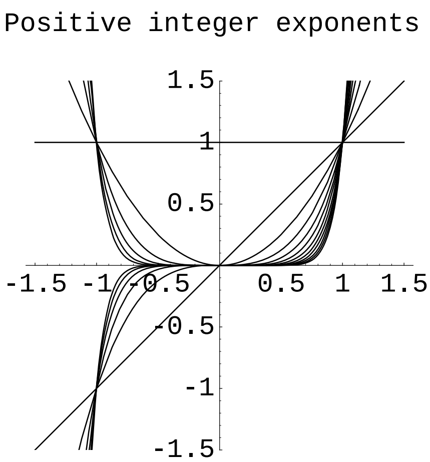
To animate the graphs, double-click on the first plot.



- *The graphs together*



• Powers with positive integer exponents together



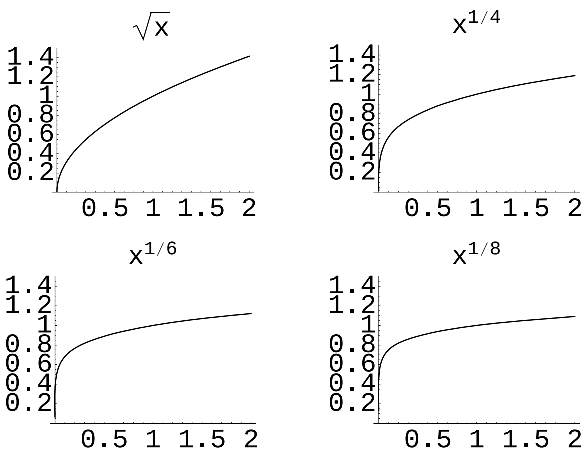
• Statements of the plots (electronic version only)

Powers with exponent $\frac{1}{2k}$ - even roots

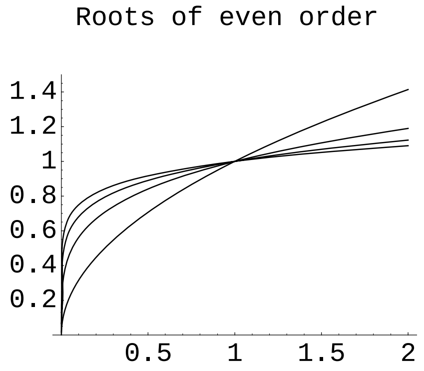
• Definition

If $x \geq 0$, then $x^{\frac{1}{2k}} := \sqrt[2k]{x}$ is the nonnegative number for which $\left(\sqrt[2k]{x}\right)^{2k} = x$. See the general definition of inverse functions.

• Graphs



- *Animation (elecronic version only)*
- *The graphs together*



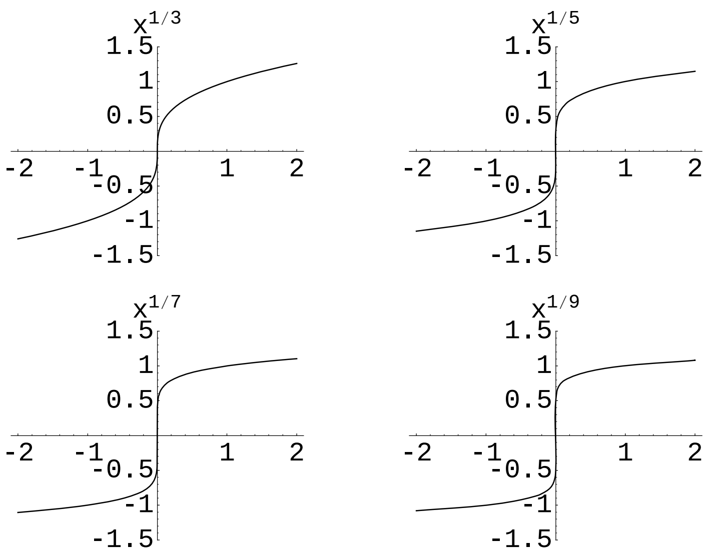
- **Statements of the plots (electronic version only)**

Powers with exponent $\frac{1}{2k+1}$ - odd roots

- **Definition**

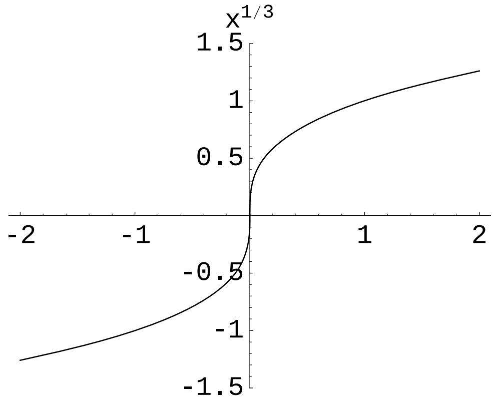
If $x \in \mathbb{R}$, then $x^{\frac{1}{2k+1}} := \sqrt[2k+1]{x}$ is that real number for which $\left(\sqrt[2k+1]{x}\right)^{2k+1} = x$. See the general definition of inverse functions.

- **Graphs**

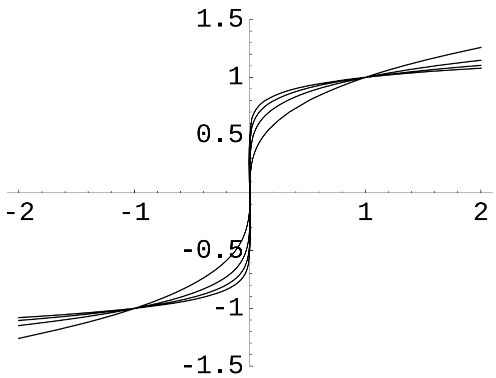


- *Animation (electronic version only)*

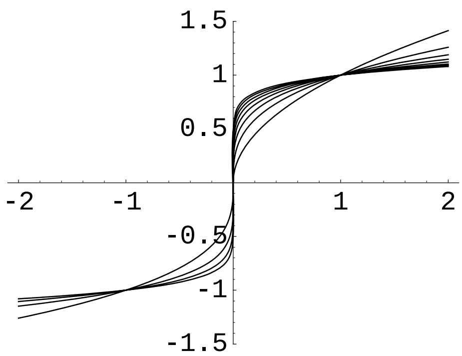
To animate the graphs, double-click on the first plot.



- *The graphs together*



- **Root functions together**



- **Statements of the plots (electronic version only)**

General nonnegative powers

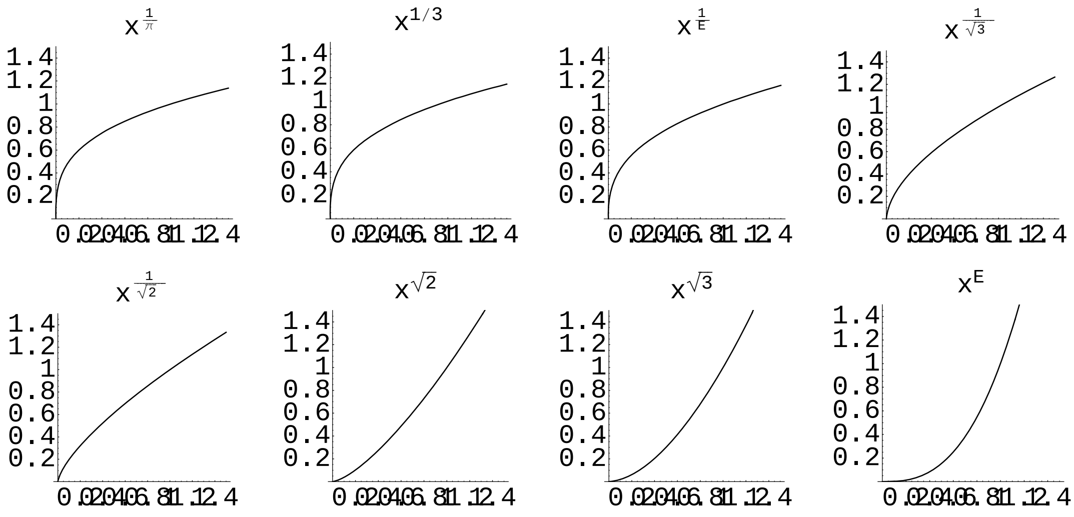
- **Definition**

The definition for rational numbers is given. For an irrational number α, the power x^α is defined by the limit

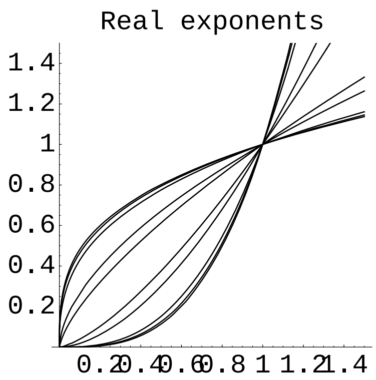
$$x^\alpha := \lim_{n\to\infty} x^{\alpha_n},$$

where $\{\alpha_n\}$ is a rational approximation of α.

• Graphs



- *Animation (electronic version only)*
- *The graphs together*



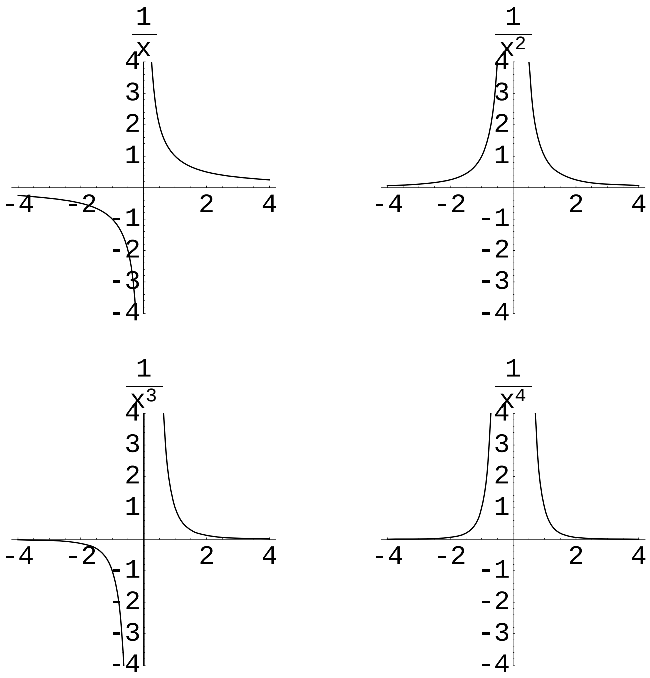
• Statements of the plots (electronic version only)

Powers with negative exponents

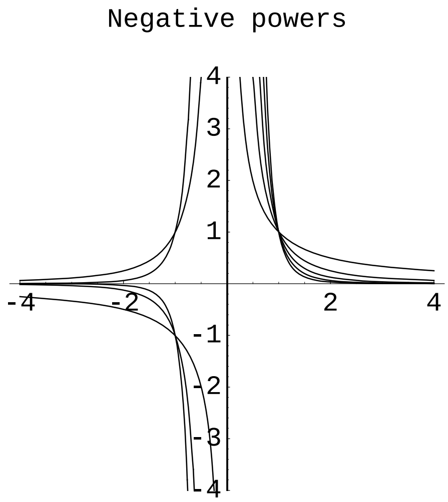
• Definition

If $x \in \mathbb{R}$, $x \neq 0$, then $x^{-\alpha} := \frac{1}{x^{\alpha}}$.

- **Graphs**



- *Animation (electronic version only)*
- *The graphs together*



- **Statements of the plots (electronic version only)**