Content

# Problems in Mathematics
# &
# Experiments with Mathematica

## 2. Elementary functions

### 2.2 Basic properties of functions

**Theory**

- **The definition of functions**

A function from the set $A$ to set $B$ is a rule that assigns to each element $x \in A$ a unique element $y \in B$. The function is denoted $f: A \to B$ to emphasize it as a rule, and the notation $y=f(x)$ is used to indicate that $y \in B$ is the value assigned to the element $x \in A$ by the function $f$. The set $A$ is the domain (denoted by $D_f$) of the function and the set of all values $R_f := \{f(x) : x \in A\}$ is the range of the function.
The set $G_f := \{(x,f(x)) \in A \times B : x \in A\}$ is the graph of the function.

- **One-to-one functions**

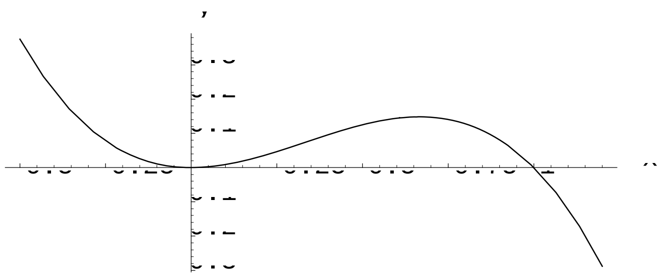
A function $f: A \to B$ is called one-to-one if to each element $y$ of the range of $f$ there exists one and only one element $x \in A$ such that $y=f(x)$, that is if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.

- **Inverse functions**

If the function $f(x)$ is one-to-one, then there exists a function $g$ for which $g(f(x))=x$ . This function is called the inverse function to $f(x)$ and denoted by $f^{-1}(x)$ or $\overline{f}(x)$.

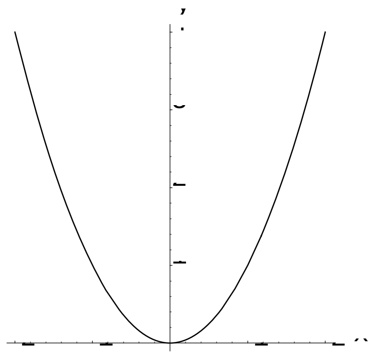
- **Real functions**

A function $f: \mathbb{R} \to \mathbb{R}$ is called scalar or real function, and
$D_f \subset \mathbb{R}$; $R_f \subset \mathbb{R}$; $G_f \subset \mathbb{R} \times \mathbb{R}$.



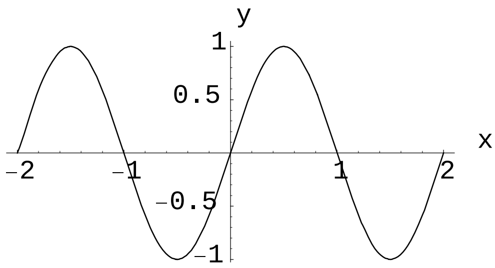
- **Properties of real functions**
  - *Even functions*

A function $f: \mathbb{R} \to \mathbb{R}$ is even if $f(-x)=f(x)$.



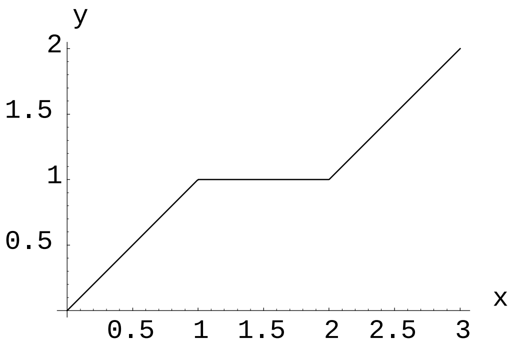
- *Odd functions*

A function $f: \mathbb{R} \to \mathbb{R}$ is odd if $f(-x)=-f(x)$.



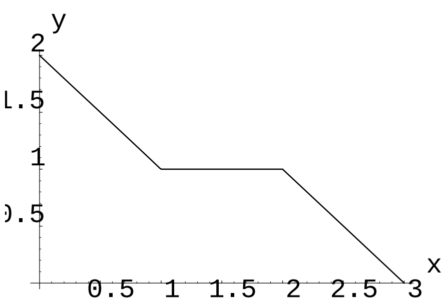

- *Nondecreasing functions*

A function $f: \mathbb{R} \to \mathbb{R}$ is nondecreasing if $x_1 < x_2$ implies $f(x_1) \le f(x_2)$.



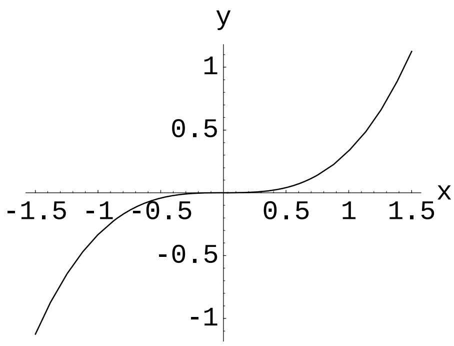

- *Nonincreasing functions*

A function $f: \mathbb{R} \to \mathbb{R}$ is nonincreasing if $x_1 < x_2$ implies $f(x_1) \ge f(x_2)$.



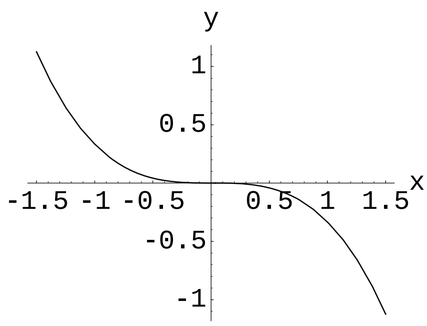

- *Increasing functions*

A function $f: \mathbb{R} \to \mathbb{R}$ is monotone increasing if $x_1 < x_2$ implies $f(x_1) < f(x_2)$.



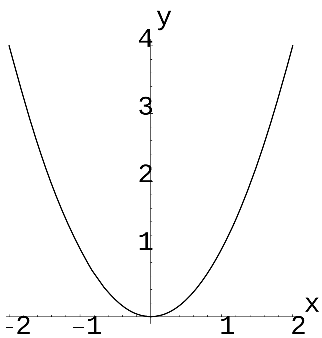

- *Decreasing functions*

A function $f\colon \mathbb{R}\to\mathbb{R}$ is monotone decreasing if $x_1 < x_2$ implies $f(x_1) > f(x_2)$.



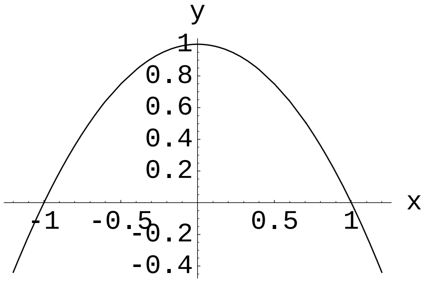

- *Local minimum*

The function $f\colon \mathbb{R}\to\mathbb{R}$ has local minimum at $x_0$ if there exists an $\epsilon>0$ such that $f(x)\geq f(x_0)$ for every $x \in (x_0-\epsilon, x_0+\epsilon)$.




- *Local maximum*

The function $f\colon \mathbb{R}\to\mathbb{R}$ has local maximum at $x_0$ if there exists an $\epsilon>0$ such that $f(x)\leq f(x_0)$ for every $x \in (x_0-\epsilon, x_0+\epsilon)$.



## *Problems and exercises*

### PROBLEM 2.2.1

Check if the following rules make functions or one-to-one functions. Define the inverse function if possible.

mother → children

mother → first child

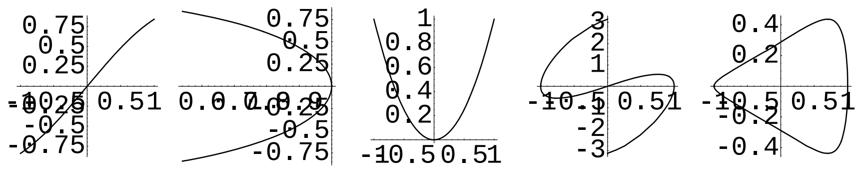
time → temperature

time → concentration of a solution

### PROBLEM 2.2.2

Find pieces of the following curves that represent graphs of functions, monotone functions. Find local maxima and minima.

(1)



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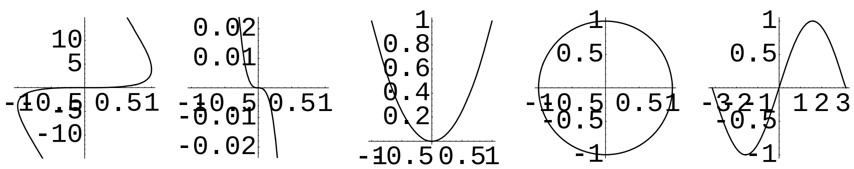

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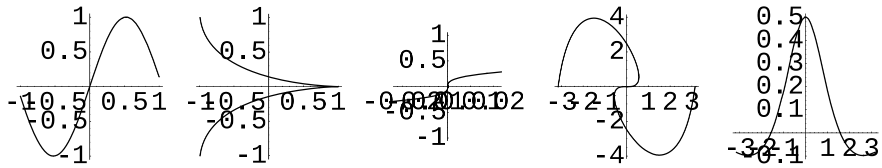

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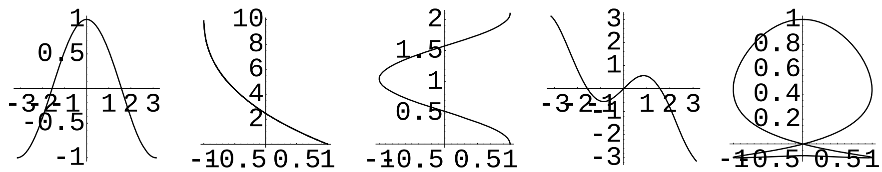

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