

Problems in Mathematics & Experiments with Mathematica

Indefinite integral

Definition, geometrical meaning, properties

Theory

Let the function $f(x)$ be defined on the interval $[a, b]$. A function $F(x)$ is an antiderivative of $f(x)$ if $F'(x)=f(x)$ for every $x \in (a, b)$.

Let the function $f(x)$ be defined on the interval $[a, b]$. If $F_1(x)$ and $F_2(x)$ are antiderivatives of $f(x)$ then $F_1(x) - F_2(x) \equiv \text{constant}$.

The collection of antiderivatives of $f(x)$ on $[a, b]$ is called indefinite integral. The notation is

$$\int f(x) dx$$

- **Geometrical meaning**

Use the geometrical meaning of the derivative. For an antiderivative $F(x)$, $F'(x) = f(x)$, i.e., it is the slope of the tangent line at $(x, F(x))$. Now, draw a little piece of a straight line of slope $f(x)$, at every (x, y) , or at least the points of a grid. Then any antiderivative has to smoothly touch the lines at the points $(x, F(x))$. We say, that it smooths to the given slope-field.

- **Find a particular antiderivative**

Let an antiderivative $F_1(x)$ be known, and let the point (x_0, y_0) be given. Find the antiderivative $F_2(x)$ for which $F_2(x_0) = y_0$. Since $F_2(x) = F_1(x) + C$, we obtain

$$F_2(x) = F_1(x) + y_0 - F_1(x_0)$$

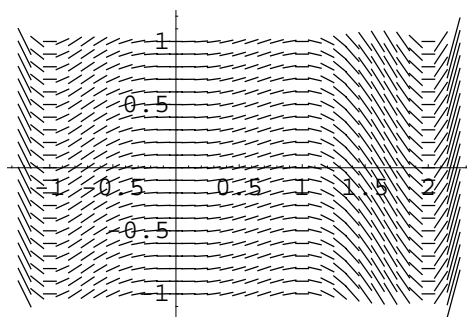
Exercises and problems

■ Mathematica initialization

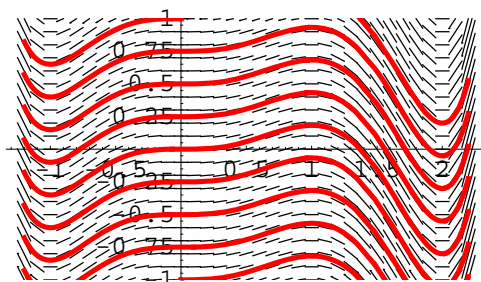
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SetDirectory["FileName" /.
NotebookInformation[EvaluationNotebook[]] /.
Which[
    $System == "Microsoft Windows",
        FrontEnd`FileName[{_, d___}, nam_, ___] :>
ToFileName[{d}] ,
    $System != "Microsoft Windows", FrontEnd`FileName[
        d_List, nam_, ___] :> ToFileName[d] ]];
<< "package//slopeprt.m";
Abs'[x_] := Which[x < 0, -1, x > 0, 1, True, 0];
Sign'[x_] := 0 ;
<< package//npow.m;
```

SOLVED PROBLEM 5.1.1 Graphical finding of antiderivatives

Sketch the graph of some antiderivatives of the given slope-field:



○ SOLUTION



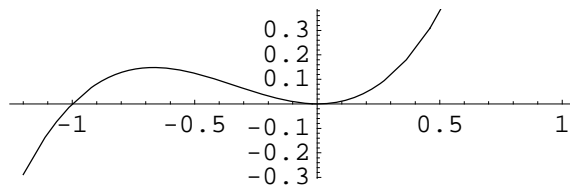
- Generating the slopefield
- Plot some Antiderivatives



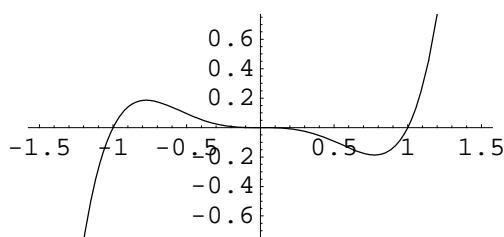
PROBLEM 5.1.2

Using the properties of the derivative, sketch the graph of an antiderivative of the following functions.

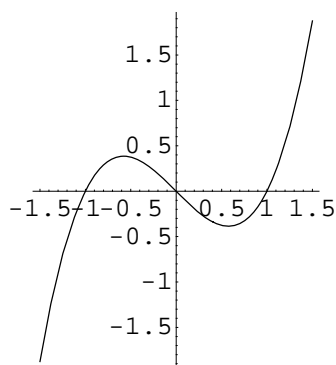
(1)



(2)



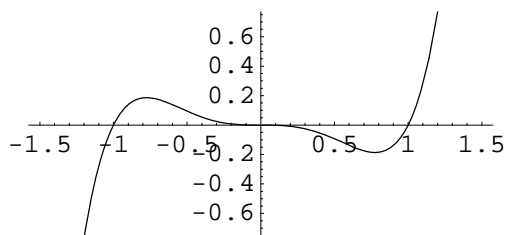
(3)



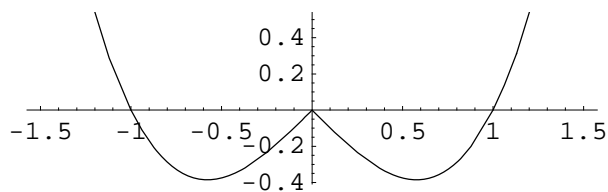
PROBLEM 5.1.3

Using the properties of the derivative, sketch the graph of an antiderivative of the following functions.

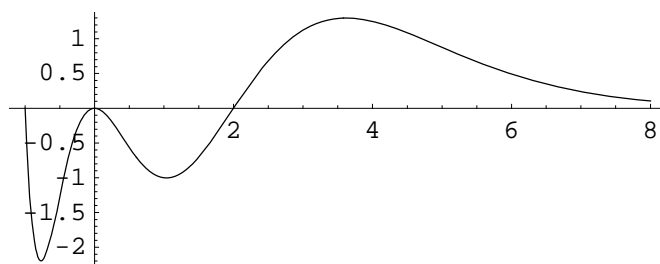
(1)



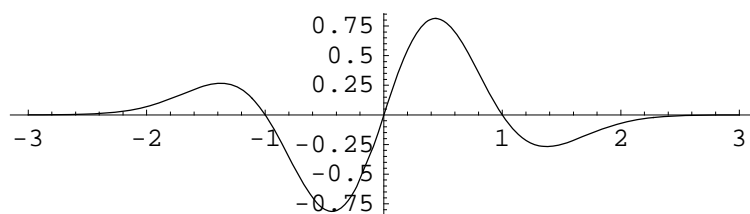
(2)



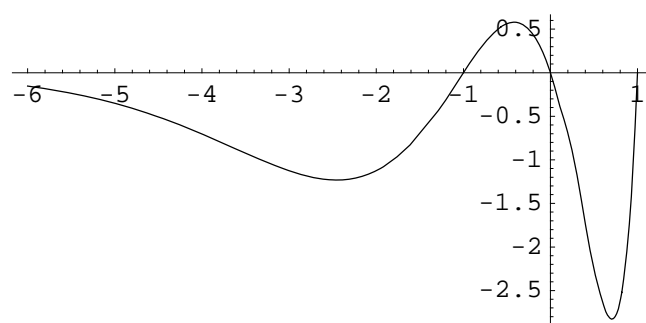
(3)



(4)



(5)



(6)

