Content

Problems in Mathematics
&
Experiments with Mathematica

5. Indefinite integral

5.3 Find a particular antiderivative

Theory

- **Method**

Let an antiderivative $F_1(x)$ be known, and let the point (x_0, y_0) be given. Find the antiderivative $F_2(x)$ for which $F_2(x_0) = y_0$. Since $F_2(x)= F_1(x)+C$, we obtain

$$F_2(x) = F_1(x) + y_0 - F_1(x_0)$$

Exercises and problems

■ *Mathematica initialization*

```
SetDirectory["FileName" /.
    NotebookInformation[EvaluationNotebook[]] /.
    Which[
            $System == "Microsoft Windows",
                FrontEnd`FileName[{_, d___}, nam_, ___] :>
     ToFileName[{d}] ,
             $System != "Microsoft Windows", FrontEnd`FileName[
                d_List, nam_, ___]      :> ToFileName[d] ]];

<< "package//slopeprt.m"
        << package//npow.m;
Abs'[x_] := Which[x < 0, -1, x > 0 , 1, True, 0];
Sign'[x_] := 0 ;
```

SOLVED PROBLEM 5.3.1

For the given $f(x)$, find the antiderivative for which $F_2(x_0)=y_0$.

$$f(x\_) := \frac{1}{2} x^2 - 3; \{x_0, y_0\} = \{1, 2\};$$

○ Solution

An antiderivative is

$$G(\mathrm{u\_}) := \int f(v)\, dv \,/.\, v \to u$$

Now, the needed one is

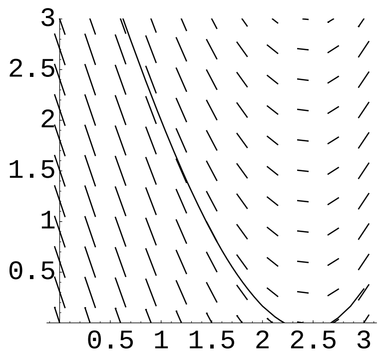
```
F[u_] := G[u] + y0 - G[x0]
```

Plot the slope portrait and *F(x)*:

```
slp = SlopePortrait[{1, f[x]}, {x, 0., 3., 0.3}, {y, 0, 3, 0.3}];

plt = Plot[F[x], {x, 0, 3},
        DisplayFunction -> Identity];

plt1 = Show[Graphics[slp], plt,
                    DisplayFunction -> $DisplayFunction,
               AspectRatio → Automatic,
     Axes → True, PlotRange -> {0, 3}];
```



Problem 5.3.2

The speed of a process *F(t)* is the known function *v(t)*. Find *F(t)* if $F(t_0) = F_0$.

(1) $t_0 = 0;\ F_0 = 0.1;\ v(\mathrm{t\_}) := 2^t;$

(2) $t_0 = 0;\ F_0 = 2;\ v(\mathrm{t\_}) := 3\, 2^{-t};$

(3) $t_0 = 0;\ F_0 = 1;\ v(\mathrm{t\_}) := \log_2(t+1);$

(4) $t_0 = 0;\ F_0 = 1;\ v(\mathrm{t\_}) := \sin(3\, t);$

(5) $t_0 = 2;\ F_0 = 3;\ v(\mathrm{t\_}) := t\, e^{-t^2};$

Problem 5.3.3 Motion of a body

The acceleration of a motion *s(t)* is the known function *a(t)*. Find *s(t)* if $s(t_0) = s_0$ and $v(t_0) = v_0$.

(1) $t_0 = 0;\ v_0 = 0.1;\ \ s_0 = 1;\ \ \ a(\text{t\_}) := 2^{-t};$

(2) $t_0 = 0;\ v_0 = 0;\ \ s_0 = 2;\ \ \ a(\text{t\_}) := 2;$

(3) $t_0 = 0;\ v_0 = 1;\ \ s_0 = 1;\ \ \ a(\text{t\_}) := \sin(t);$

(4) $t_0 = 2;\ v_0 = 2;\ \ s_0 = 0;\ \ \ a(\text{t\_}) := 1 - 3^{-t};$