Content

Problems in Mathematics & Experiments with Mathematica

2. Elementary functions

2.9 Logarithmic transformations

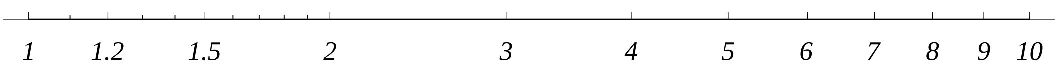
Theory

- **Logarithmic scales**

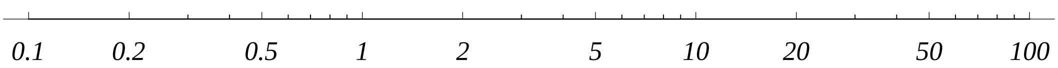
Consider a line with given origin and a uniform scaling for real numbers. Let the number $(a>1)$ be given. Taking a real number $x>0$, mark it at $z = \log_a x$ and label the point with x. We obtain a logarithmic scale for positive numbers. In particular, label the ticks at $z = \log_a x = k = 0, \pm 1, \pm 2, \ldots$ with a^k, i.e., $\ldots,\ -2 \to a^{-2},\ -1 \to a^{-1},\ 0 \to 1,\ 1 \to a,\ 2 \to a^2,\ \ldots$. Consequently, the distances between the ticks a^k and a^{k+1} are equal.

- *Special case* $(\lg x,\ a = 10)$:

The interval $[1, 10]$:



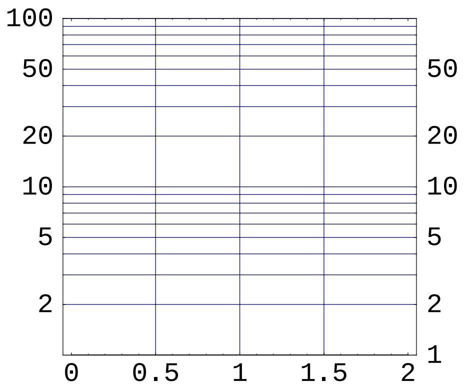
The interval $[0.1, 100]$:



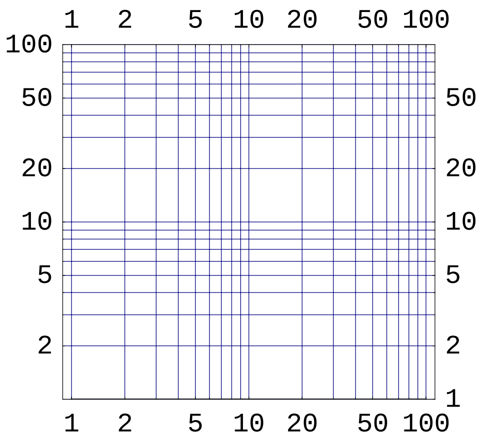
- **Logarithmic coordinate systems**

Let us use logarithmic scaling for one or both axes of the coordinate system. Then we have a logarithmic or double logarithmic coordinate system.

- *Logarithmic system: y-axis is logarithmically transformed*



- *Double logarithmic system: both axes are logarithmically transformed*



• Logarithmic transformation and logarithmic plot

Let *(x,y)* be a point in the plane, where *y>0*. The transformation

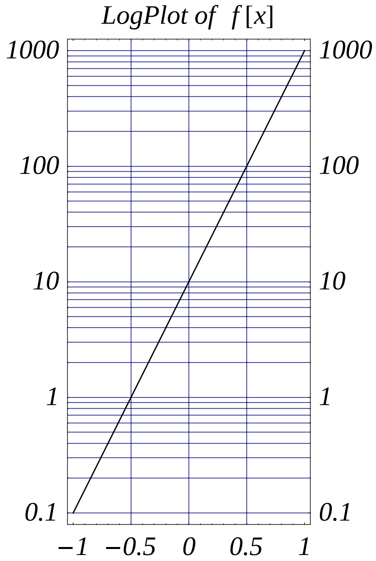
$$(x,\ y) \to (x,\ z) := (x,\ \log_a y)$$

is called logarithmic transformation. The point *(x,z)* can be plotted in the *xz*-coordinate system. This gives the same result, as the point is plotted in logarithmic coordinate system. This latter called logarithmic plot.

Let *y=f(x)>0*. The transformation $z = \log_a f(x)$ is the logarithmic transformation for the function *f(x)*. Now, *z* can be plotted in the *xz*-coordinate system. Plotting the function immediately in the logarithmic coordinate system (logarithmic plot of the function) results in the same curve.

In particular, if $y_i \approx A\, a^{b\, x_i}$, then using the logarithmic transformation, the transformed data set is $z_i = b\, x_i + \log_a A$, i.e., the graph is a straight line in the *xz*-coordinate system.

- *Example: the logarithmic plot of* 10^{2x+1}.



• Double logarithmic transformation and double logarithmic plot

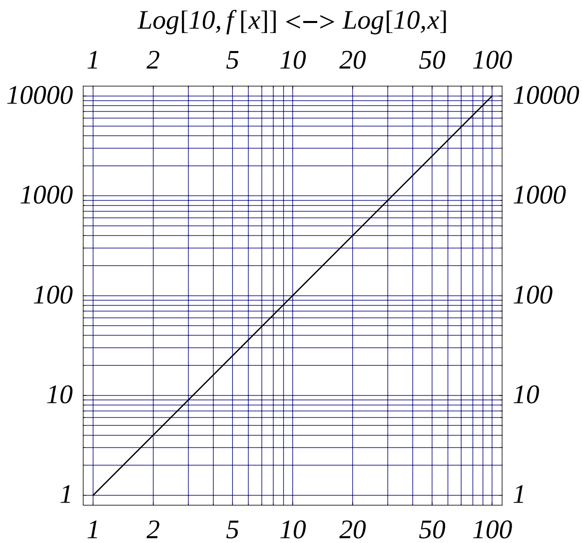
Let *(x,y)* be a point in the plane, where x>0, *y>0*. The transformation

$$(x, y) \to (u, z) := (\log_a x, \log_a y)$$

is called double logarithmic transformation. The point *(u,z)* can be plotted in the *uz*-coordinate system. This plot yields the same result, as if the point was plotted in double logarithmic coordinate system. The latter is called double logarithmic plot.

Let *y= f(x)>0* and x>0. Take $z = \log_a f(x)$. Introduce the new variable $u = \log_a x$. Plotting z in the*(u,z)*-coordinate system and also plotting *f(x)* in the logarithmically scaled *xy*-coordinate system give the same curve.
In particular, if $y_i \approx A\, x_i^b$, then using this transformation, the transformed data set is $z_i = b \log_a x_i + \log_a A = b\, u_i + \log_a A$, i.e., it is a straight line in the *uz*-plane.

- *Example : the double logarithmic plot of* x^2



Mathematica summary

• Statement summary

The logarithmic plot statements belong to the package `Graphics`Graphics``.

`LogPlot[...]`	$z = \lg(f(x))$ is plotted, logarithmic scaling for y.
`LogLogPLot[...]`	$z = \lg(f(x))$ is plotted as a function of $u = \lg(x)$, logarithmic scaling for both axes.
`LogListPlot[...]`	$(x, z) = (x, \lg y)$ is plotted, $y-$ axis is logarithmically scaled.
`LogLogListPlot`	$(u, z) = (\lg x, \lg y)$ is plotted. Both axes are logarithmically scaled.
`UnitScale[...]`	Normal scale
`LogScale[...]`	Logarithmic scale (lg scale !)

• Typical Plot statements

```
Needs["Graphics`Graphics`"]
```

- *Logarithmic data plot*

```
LogListPlot[data, AspectRatio → Automatic,
  GridLines → Automatic, Frame → True,
                 PlotStyle -> {PointSize[0.03]}];
```

- *Double logarithmic data plot*

```
LogLogListPlot[data, AspectRatio → Automatic,
  GridLines → Automatic, Frame → True,
                 PlotStyle -> {PointSize[0.05]}];
```

- *Logarithmic plot of functions*

```
LogPlot[f[x], {x, x0, x1}, AspectRatio → Automatic,
  GridLines → Automatic, Frame → True];
```

- *Double logarithmic plot of functions*

```
LogLogPlot[f[x], {x, x0, x1}, AspectRatio → Automatic,
  GridLines → Automatic, Frame → True];
```

Exercises and problems

■ *Mathematica initialization*

```
Needs["Graphics`Graphics`"]
```

SOLVED PROBLEM 2.9.1 Logarithmic data plot

Plot the following points in logarithmic coordinate system.

```
data = {{1, 1}, {0, 0.1}, {0.5, 5}, {1.2, 10}, {3, 12}};
```

○ SOLUTION

• *Plotting manually*

Perform the transformation $(x, z) = (x, \lg y)$:

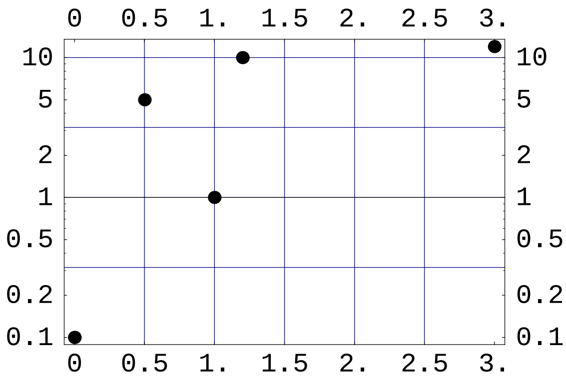
```
logdata = data /. {x_, y_} -> {x, Log[10, y]}
```

Or, one can use

```
logdata =
 Transpose[{Transpose[data][[1]], Log[10, Transpose[data][[2]]]}]
```

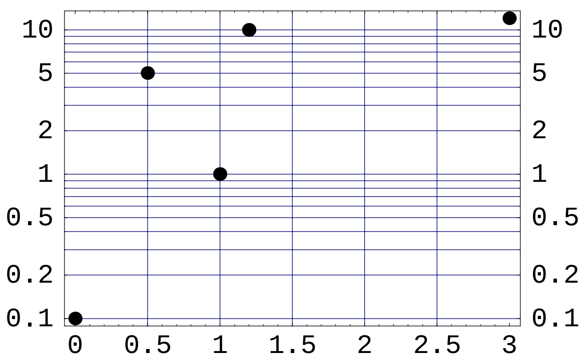
Plot z in the xz-plane, and use logarithmic scaling for the z-axis:

```
dlogplt = ListPlot[logdata, Frame → True,
   FrameTicks -> {UnitScale[#1, #2, 0.5, 10] &, LogScale},
   GridLines -> Automatic, AspectRatio -> Automatic,
        PlotStyle -> {PointSize[0.03]}];
```



• *Using the LogListPlot statement*

```
sdlogplt = LogListPlot[data, AspectRatio → Automatic,
   GridLines → Automatic, Frame → True,
                 PlotStyle -> {PointSize[0.03]}];
```



○

PROBLEM 2.9.2

Plot the following points in a logarithmic coordinate system.

```
(1) data = {{0, 2}, {0.5, 0.2}, {0.5, 1.5}, {2.2, 10}, {3.5, 15}};
```

```
(2) data = {{1, 1.5}, {0, 0.5}, {0.6, 5.5}, {1.5, 10}, {3, 1}};
```

```
(3) data = {{0.1, 1}, {0.5, 0.2}, {0.5, 3}, {1.2, 12}, {3, 24}};
```

Solved Problem 2.9.3 Double logarithmic data plot

Plot the following points in a double logarithmic coordinate system.

```
data = {{1, 1}, {10, 12}, {0.1, 3}, {15, 0.1}, {10, 14}};
```

○ Solution

• *Plotting manually*

Perform the transformation $(x, z) = (\lg x, \lg y)$.

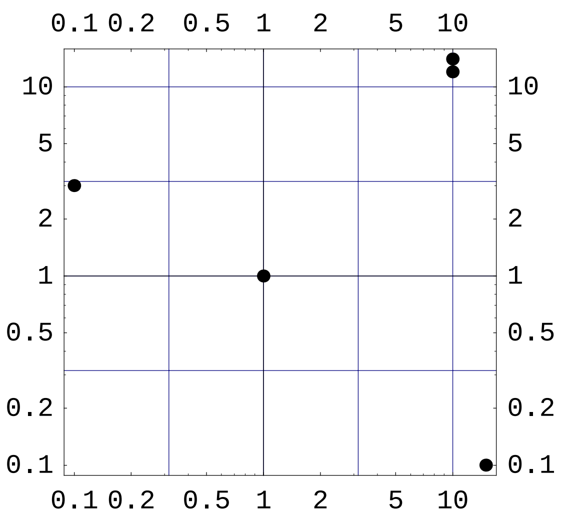
```
llogdata = data /. {x_, y_} -> {Log[10, x], Log[10, y]}
```

Or, one can use

```
llogdata = Transpose[{Log[10, Transpose[data][[1]]],
                      Log[10, Transpose[data][[2]]]}]
```

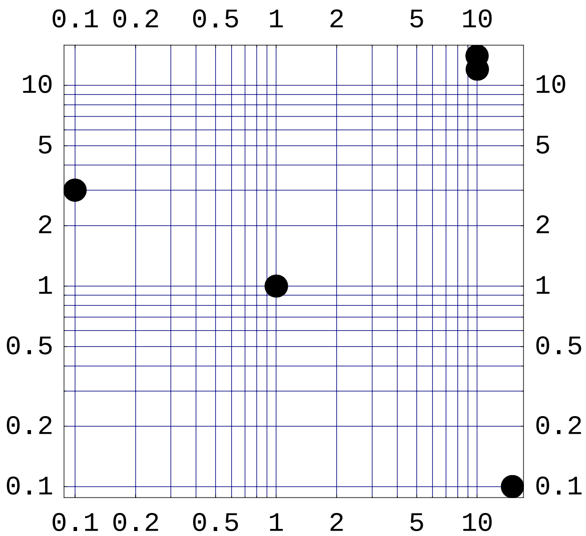
Plot z in the xz-plane, and label the z-axis with logarithmic scale:

```
dllogplt = ListPlot[llogdata,
   Frame → True, FrameTicks -> {LogScale, LogScale},
   GridLines -> Automatic, AspectRatio -> Automatic,
        PlotStyle -> {PointSize[0.03]}];
```



- *Using the LogLogListPlot statement*

```
sdllogplt = LogLogListPlot[data,
   AspectRatio → Automatic, GridLines → Automatic, Frame → True,
                 PlotStyle -> {PointSize[0.05]}];
```



PROBLEM 2.9.4

Plot the following points in double logarithmic coordinate system.

(1) `data = {{0, 2}, {0.5, 0.2}, {0.5, 1.5}, {2.2, 10}, {3.5, 15}}`

(2) `data = {{1, 1.5}, {0, 0.5}, {0.6, 5.5}, {1.5, 10}, {3, 1}}`

(3) `data = {{0.1, 1}, {0.5, 0.2}, {0.5, 3}, {1.2, 12}, {3, 24}}`

SOLVED PROBLEM 2.9.5 Logarithmic plot

Plot the following function in a logarithmic coordinate system on the interval $[x_0, x_1]$:

$$f(x\_) := 10^{\frac{x}{2}+1};\ x_0 = -1;\ x_1 = 2;$$

○ SOLUTION

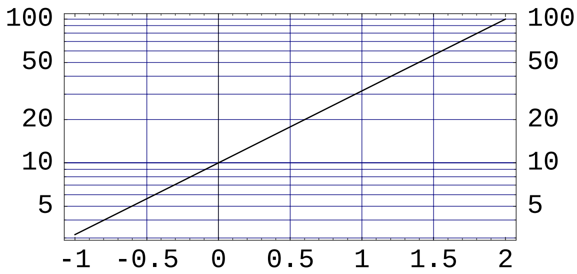
- *Plotting manually*

Perform the transformation $z = \lg(f(x))$.

```
z = Log[10, f[x]];
```

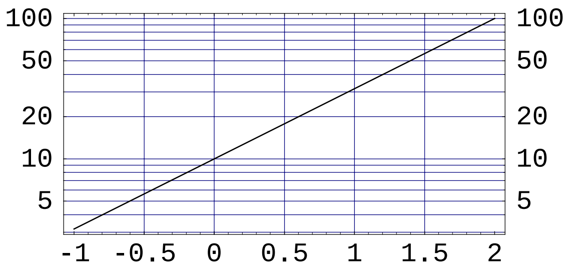
Plot z in the xz-plane, and use logarithmic scale for the z-axis:

```
loggrid = Join[Table[Log[10, i], {i, 1, 10}],
         Table[Log[10, i], {i, 1, 10}] + 1];
logplt = Plot[z, {x, x0, x1},
    Frame → True, FrameTicks -> {Automatic, LogScale},
    GridLines -> {Automatic, loggrid}, AspectRatio -> Automatic];
```



- *Using the LogPlot statement*

```
slogplt = LogPlot[f[x], {x, x0, x1},
    AspectRatio → Automatic, GridLines → Automatic, Frame → True];
```



PROBLEM 2.9.6

Plot the following functions in a logarithmic coordinate system.

10^{2-3x}; 10^{x-3}; $10^{\sin x}$; 10^{-x^2}; 3^{x-2}; $2^{\tan(x)}$; x^3; $3x^4$;

SOLVED PROBLEM 2.9.7 Double logarithmic plot

Plot the following function in double logarithmic coordinate system on the interval $[x_0, x_1]$:

f(x_) := $\sqrt{x/2}$; $x_0 = 1$; $x_1 = 100$;

○ SOLUTION

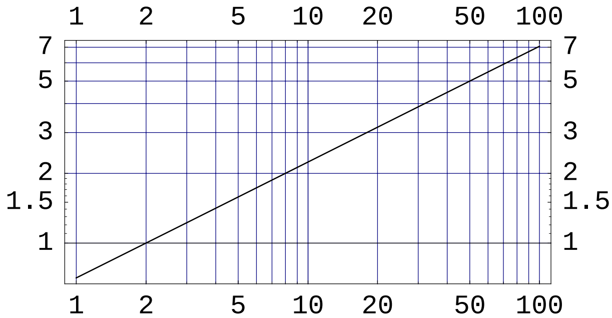
- *Plotting manually*

Perform the transformation $z = \lg(f(x))$ and $u = \lg(x)$:

```
z = Log[10, f[x]] /. x -> 10^u;
 u0 = Log[10, x0]; u1 = Log[10, x1];
```

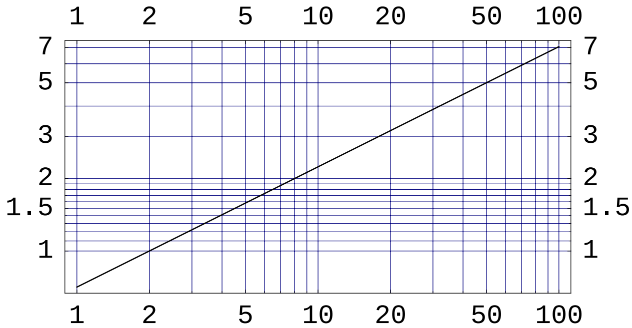
Plot (u,z) in the uz-plane, and label the z-axis with logarithmic scale:

```
loggrid = Join[Table[Log[10, i], {i, 1, 10}],
        Table[Log[10, i], {i, 1, 10}] + 1];
llogplt = Plot[z, {u, u0, u}, Frame → True,
        FrameTicks -> {LogScale, LogScale},
   GridLines -> {loggrid, loggrid}, AspectRatio -> Automatic];
```



- *Using the LogLogPlot statement.*

```
sllogplt = LogLogPlot[f[x], {x, x0, x1},
   AspectRatio → Automatic, GridLines → Automatic, Frame → True];
```



PROBLEM 2.9.8

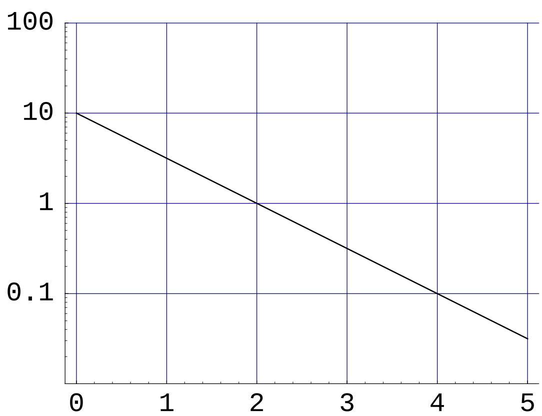
Plot the following functions in a double logarithmic coordinate system.

10^{1-2x}; 3^{x-1}; $10^{1-\log(x)\,x}$; 10^{-x^2}; 3^{x-2}; $2^{\tan(x)}$; x^5; $4x^2$;

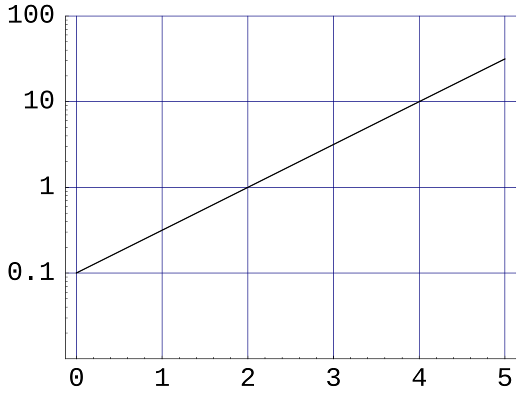
PROBLEM 2.9.9

Regognize the formulas of the functions by the following logarithmic graphs:

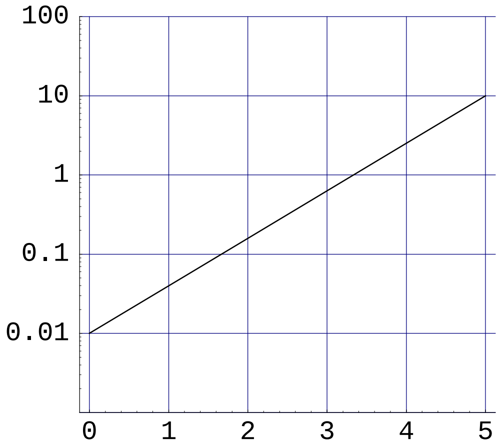
(1)



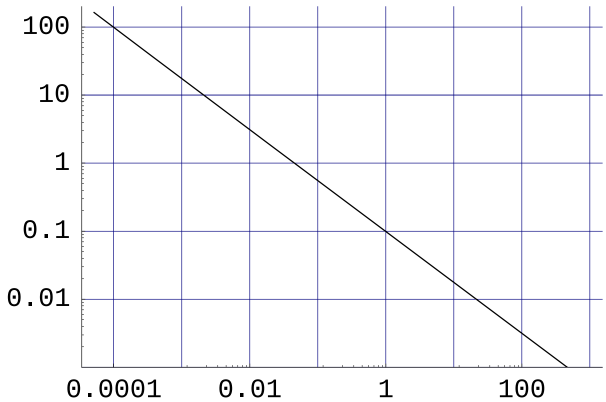
(2)


100
10
1
0.1
0
1
2
3
4
5

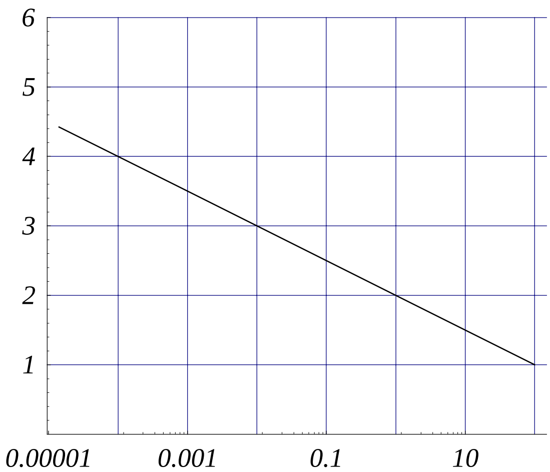
(3)


100
10
1
0.1
0.01
0
1
2
3
4
5

(4)


100
10
1
0.1
0.01
0.0001
0.01
1
100

(5)


6
5
4
3
2
1
0.00001
0.001
0.1
10