



*Jó szomszédok a közös jövőért
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University of Szeged - UNS Faculty of Science Novi Sad

Teaching Mathematics and Statistics in Sciences HU-SRB/0901/221/088

Abstracts

for the
GeoGebra Conference

International Conference of Teaching and Learning Mathematics (ICTLM-2011)

(Međunarodna Konferencija o nastavi i učenju matematike)

Novi Sad, 15-16, January, 2011

Novi Sad, Srbija

January 15-16, 2011



A program a Magyarország – Szerbia IPA Határon Átnyúló Együttműködési Programban az Európai Unió társfinanszírozásával valósul meg.
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Program of the **International GeoGebra Conference** for Southeast Europe January 15-16, Novi Sad, Serbia

Saturday, January 15th, 2011

	Saturday, January 15th, 2011	
9:00 – 9:30	Conference opening – A1 (ground floor)	
9:30 – 10:00	Zsolt Lavicza Introducing GeoGebra and its future developments	
10:30 – 11:00	Zoltan Kovács What's new in GeoGebra	
11:00 – 11:30	Đurđica Takači GeoGebra and fractional derivatives	
11:30 – 12:00	Judit Pacsirta GeoGebra Institutes, Conferenes and News	
12:00 – 12:30	Toni Chehlarova Creating of specific learning environments within <i>GeoGebra</i>	
12:30 – 13:00	Zlatko Udovičić On the polynomial approximations	
13:00 – 14:30	LUNCH (Student restaurant, 1st floor)	
	Geogebra with the Seminar chair: Đurđica Takači, Matija Lokar A1 (Serbian language)	Dynamic geometry chair: Janos Karsai, Arpad Takači (English language)



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14:30 – 15:00	Matija Lokar Using Simple Video Tutorials in Teaching with Geogebra	Janos Karsai Modeling the fight of species for territory with cellular automata
15:00 – 15:30	Dessislava Dimkova Exploring and discovering mathematics with dynamic worksheets	Nárcisz Kulcsár, Lajos Szilassi Special locuses
15:30 – 16:00	Mira Jovanović Matematičko modeliranje i GeoGebra	Marina Milovanović Multimedia Approach in Teaching Mathematics
16:00 – 16:30	Šime Šuljić GeoGebra u nastavi matematike	Davorka Radaković Razvoj softvera za dinamičku geometriju na Silverlight platformi
16:30 – 17:00	Elvira Šipoš Izometrijske transformacije u GeoGebra DGS	Zlatan Magajna Detecting patterns and properties in Geogebra constructions
17:00 – 17:30	Nenad Radaković Using GeoGebra for teaching and learning probability	Marina Petrović Testiranje i provera znanja na Moodle sistemu
17:30 – 18:00		Aleksandar Bukva, Živko Raičević Kinematika kosog hica
17:40 – 18:40	ROUND TABLE Experiences from: Bosnia and Herzegovina, Bulgaria, Croatia, Iran, Serbia Josip Kličinović: Croatian experience in using GeoGebra	
20:00	CONFERENCE DINNER (Hotel Sajam, Zlatna medalja) All participants are invited	

Sunday, January 16th, 2011

9:00 – 9:30	Academician Petar S. Kenderov The Role of European Project "Fibonacci" for the New Interplay between Science and Education	
9:30 – 10:00	Evgenia Sendova The beauty in mathematics and the mathematics in the beautiful	
10:30 – 11:00	Matija Lokar A New Teacher's Role and e-resources	
11:00 – 11:30	COFFEE BREAK	
11:30 – 12:00	Acad. Petar Kenderov, Evgenia Sendova, Toni Chehlarova, Dessi Dimkova Workshop Inquiry-based learning within dynamic mathematics environments: evolutionary and revolutionary changes in mathematics education	Peter Kortesi Teaching functions
12:00 – 12:30		Lila Korenova
12:30 – 13:00		Duška Pešić Epsilon-delta definicija neprekidne funkcije
13:00 – 13:20	Iordanka Gortcheva Demonstrative Approach to Ancient Constructive Problems through GeoGebra	Jelena Tatar Transformacije grafika funkcije
13:20 – 13:40	Tanja Sekulić Geogebra i matematičko modelovanje u nastavi matematike	Abdul-Sahib Hasani N. Plane Mirror in GeoGebra Laboratory



13:40 – 14:30	BUFFET LUNCH (at the DMI)	
14:30 – 14:50	Ana Donevska Todorova Computer Algebra Systems Supporting Teaching/Learning Linear Algebra	Đorđe Herceg Upotreba parametara u geometrijskim konstrukcijama u GeoGebri
14:50 – 15:10	Zoran Trifunov, Linda Fahlberg, Stojanovska Learning Trigonometric Functions Using GeoGebra	Kaja Maričić Problem Solving Using GeoGebra
15:10 – 15:30	Zoran Trifunov Learning Trigonometric Functions Using GeoGebra	Eugen Ljajko, Vait Ibro Development of ideas in a GeoGebra – aided mathematics instruction
15:30 – 15:50	Miljana Ariconović Kompleksni brojevi u programu <i>Geogebra</i>	Milena Marić Using GeoGebra and Modern Web Technologies for Teaching Analytic Geometry
15:50 – 16:10	Danka Lučić Grafički prikaz trigonometrijskih funkcija	Zoran Trifunov, Igor Dimovski Vector-Valued Functions and Geogebra
16:10 – 16:30	Tijana Stojančević Logarithmic function and GeoGebra	Mario Varga Ekstremne vrednosti funkcije i primene
16:30	Conference closing	



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The Role of European Project “Fibonacci” for the New Interplay between Science and Education

Petar S. Kenderov

Institute of Mathematics and Informatics
Bulgarian Academy of Sciences

Until 70-80 years ago a good university education was enough to ensure lifelong successful career without significant additional training. Today the situation is quite different. Within 5 -10 years after completion of university education young professionals may encounter in their daily work problems that have not been studied in school or at university. Knowledge and technology are changing so fast that even the idea of further education through “Lifelong Learning” cannot help. Many innovations quickly bring profits and quickly fade away, much before they reach in-service or pre-service educational institutions. The only reasonable way for an expert to “keep up to date” is to be able to:

- identify (feel, notice) the new problems and new developments in relevant areas of science and technology;
- fill in gaps in his/her knowledge by using appropriate sources of information (scientific literature, internet, professional societies, social networks, etc.);
- realize the essence and provide innovative and reasonable solution to the encountered problems;
- present convincing arguments in favor of the proposed solution.

In short, the expert must be able to conduct own exploration, apply creatively the findings and present the results in a way convincing for other people. Such abilities are gradually becoming part of the skills that are expected from the young university graduates. It is not an exaggeration to say that these abilities of the graduates distinguish good from mediocre universities.

Like any other skills, ability and inclination to investigation can be developed and cultivated through proper education. Such education should include elements of research and innovations. To make this happen it is necessary to transform the now dominant educational style of teaching facts, that are passively absorbed by students, to a style in which students actively participate in the learning process by experimenting, asking questions, conjecturing, providing arguments and, finally, by self-discovering the studied facts and the interconnections between them. The knowledge acquired this way is more functional and more deeply nested in the minds and souls of students. Due to the existence of software systems like GEOGEBRA, GEONExT, ELICA, CABRY, SKETCHPAD etc., this change in the style is feasible in mathematics education as well. The European Project **FIBONACCI**, the major goal of which is just such a change will be presented and discussed.





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The beauty in mathematics and the mathematics in the beautiful or how to reveal the real spirit of mathematics to innovative teachers

Evgenia Sendova
Institute of Mathematics and Informatics
Bulgarian Academy of Sciences

The paper deals with implementing an educational strategy developed in the frames of the *I*Teach (Innovative Teacher)* European project and further implemented and elaborated in three other projects dealing with learner-centered and inquiry-based methods in math education – *Math2Earth*, *InnoMathEd* and *Fibonacci*. A possible approach to acquiring *enhanced ICT skills* specified as the synergy between technical and soft skills is presented in the context of ICT qualification courses and *InnoMathEd* and *Fibonacci* teacher training courses. Some projects developed and presented by the participants having worked on the theme *The beauty in mathematics and the mathematics in the beautiful* are discussed. The experience gained confirms our belief that the learners construct new knowledge with particular effectiveness when they are engaged in constructing something that is meaningful to themselves and to others around them.

Introducing GeoGebra and its future developments

Zsolt Lavicza,
University of Cambridge, UK

GeoGebra is rapidly gaining popularity in the teaching and learning of mathematics around the world. Currently, GeoGebra is translated to 55 languages, used in 190 countries, and downloaded by approximately 300,000 users in each month. This increased use compelled the establishment of the International GeoGebra Institute (IGI) that serves as a virtual organization to support local GeoGebra initiatives and institutes. There are already 49 established institutes on every continent, which pursue training and support of teachers, develop teaching materials, and carry out research projects. In my talk, I will review examples of applications of GeoGebra at different levels of education, introduce various lines of research with GeoGebra, and outline the future of software and community developments.

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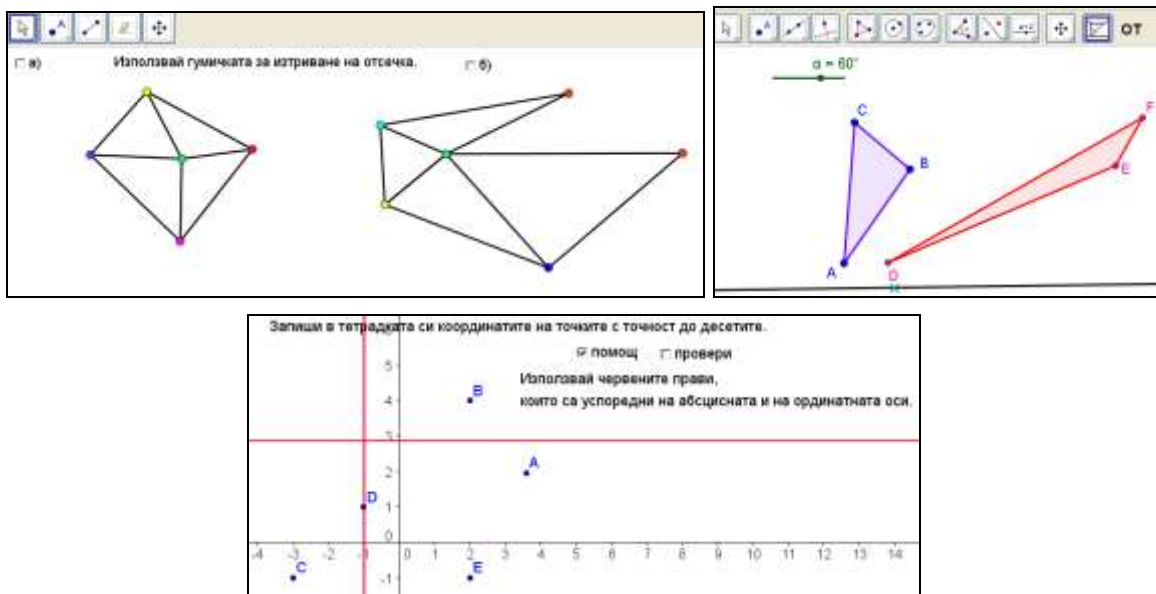
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Creating of specific learning environments within *GeoGebra*

Toni Chehlarova

Institute of Mathematics and Informatics
Bulgarian Academy of Sciences

Some opportunities for working in a specially modeled learning environment by means of *Geogebra* are presented – by reducing the tools, by creating new tools (buttons) or specific other instruments (e.g. moving lines).



Some ideas about providing information under certain (e.g. logical) conditions, including graduated help on learner's request are considered.

These ideas are illustrated by specific learning environment developed for mathematic education.

Upotreba parametara u geometrijskim konstrukcijama u GeoGebri

Dragoslav Herceg, Đorđe Herceg

Departman za matematiku i informatiku,
Prirodno-matematički fakultet, Univerzitet u Novom Sadu

Geometrijske konstrukcije u GeoGebri se mogu koristiti u nastavi na razne načine: kao sredstvo za demonstraciju određenih koncepata i pojmova, prilikom izrade konstrukcija, izvođenja dokaza ili prilikom generisanja personalizovanih zadataka i rešenja za testove. Upotrebom parametara, geometrijske konstrukcije se mogu animirati i menjati na jednostavan način. U izlaganju

prikazujemo nekoliko karakterističnih primera upotrebe parametara u GeoGebri, kao i nekoliko načina za kontrolu više parametara odjednom.

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Fractional calculus and Geogebra

Đurđica Takači

Department of Mathematics and Informatiku
Faculty of Sciences, University of Novi Sad

In this paper we present the use of the package *Geogebra* in visualization of the solution of differential equations, even the solution is obtain in the frame of generalized functions. The fractional integrals, fractional derivatives, the exact and approximate solutions of fractional differential equations, and the convolutions and delta function, and delta sequences are vizualized with the help of *GeoGebra*. In particular, by using sliders it can be followed the change of the power of derivative and the integral.

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Inquiry-based learning within dynamic mathematics environments: evolutionary and revolutionary changes in mathematics education

Workshop

**Petar Kenderov, Evgenia Sendova,
Toni Chehlarova, Dessi Dimkova**

The specifics of dynamic mathematics software such as *GeoGebra*, *Geonext* etc. provides opportunities for organizing of inquiry-based learning enabling students to rediscover mathematical phenomena and properties.

We have been trying to implement this approach in the context of several European projects (the most recent one being Fibonacci) expecting some evolutionary and some revolutionary changes.

The evolutionary changes are appropriate for current in-service teachers who could use ready-made teaching materials, could modify existing materials, or even could develop their own educational materials. The specifics in training such teachers is the synthesis between acquiring the necessary technical skills and the inquiry-based approach.

The revolutionary changes are directed to pre-service teachers, students, and their family environment. With this audience in mind we develop educational modules which are not closely related to the existing curriculum but reveal the real spirit of mathematics as a science, form skills for modeling, enhance the thinking and the imagination.

Combining the team- and the individual work, the learning at home and at school limits the shortcomings of these educational forms.





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Representative examples (in Bulgarian and English language) of such educational sources could be found at the website of the *Education in Mathematics and Informatics Department* of the Institute of Mathematics and Informatics at the:

<http://www.math.bas.bg/omi/InnoMathEd/archive.htm>

Modelling the fight of species for territory with cellular automata

János Karsai

University of Szeged

Simulation is the only effective tool for the study of several problems in biomedical modeling. Hence, computing and visualization tools became unavoidable. There are several – either general or special -computer software programs available designed for different kind of problems, such as Mathematica, Maple, Matlab, Modelica, Anylogic, etc.

In our talk, we consider stochastic cellular automata as tools in ecological research. After a short introduction to cellular automata, we briefly study the spatio-temporal development of some single species territory-based models. Then we consider competition of several species fighting for territory. We can find simulations can give us a lot of information on special properties such as aggregation, diffusion, role of neighbors in colonization of empty cells or extinction of the species from a cell.

Research and development are co-financed by the Hungary-Serbia IPA Cross-border Co-operation Programme in the frame of the project IPA HU-SRB/0901/221/088.

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Különleges mértani helyek

Kulcsár Nárcisz, Szilassi Lajos

Szegedi Tudományegyetem

A mértani hely (adott tulajdonságú pontok halmaza) a geometria egyik legtöbbet használt fogalma. Megítélés kérdése, hogy mikor érdemli ki ez a fogalom a „különleges” jelzőt. Legtöbbször abban az esetben, ha problémamegoldás közben sejtésünk sincs arról, hogy az adott tulajdonságú pontok milyen alakzatot írnak le, vagy a megsejtett (kapott) mértani hely nehezen leírható, nem szerkeszthető vonal.

Kihhasználva a Geogebának azt a tulajdonságát, hogy a vizuális megjelenítés háttérében egy nagyon erős matematikai (számítástechnikai) „motor” működik, bemutatunk néhány példát ilyen különleges mértani helyek előállítására, amelyek olykor egy-egy probléma szemléletessé tételét, olykor szerkesztési feladatok megoldását segítik.



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A kutatás és fejlesztés a Magyarország – Szerbia IPA Határon Átnyúló Együttműködési Programban az Európai Unió társfinanszírozásával az IPA HU-SRB/0901/221/088 projekt keretében valósul meg.

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Special locuses

Nárcisz Kulcsár, Lajos Szilassi
University of Szeged

The locus (set of points with given properties) is one of the most commonly used geometry concept. Of discretion that when this concept is worthy of a "special" attribute. Mostly in cases when we have no idea during problem solving what kind of shape the points with given properties describe or the guessed (given) locus is difficult to describe, it is not a constructible line.

Using that, GeoGebra has the advantage of the features that in the background of the visual display a very strong mathematical (computer) "engine" works, we present some examples of constructing such special locuses which help sometimes to present a problem more expressively, to solve constructional tasks.

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Teaching functions

Peter Kortesi
University of Miskolc, Hungary

The students and teachers can use GeoGebra in order to introduce, study and deepen the notion of function. The software is suitable to enhance the student in understanding the notions of compound functions, inverse functions, injections, surjection. The paper will analyse the possibilities offered by the software.

Keywords: Function, inverse of a function, compound function, surjection, injection.

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What's new in GeoGebra

Zoltan Kovacs,
University of Szeged, Hungary

There is growing demand to harness the power of the open dynamic mathematics software, GeoGebra. Thousands of new users world-wide start to learn its usage every day. But both old and new users come with new ideas and suggestions for its usability. As a consequence, it is a complex task to keep the control of the software development and not leaving the track of ease-of-use and intuitivity, by giving the users the possibility to help the software making a better tool in education. The GeoGebra developer team, lead by Markus Hohenwarter and Michael Borcherds, is committed to make GeoGebra such a community software which is suitable for high quality mathematics education with modern technology. A number of different subprojects are working on different enhancements for GeoGebra. Dozens of contributors from Europe, America and Asia, including mathematics teachers and hobby programmers, are working in their spare time or by profession to develop GeoGebra to be the swiss army knife for learning mathematics.

The talk will outline the past and the present of the GeoGebra development. It will highlight some of the upcoming features in version 4.0 which will be released this summer. There are too numerous to mention all features (it can be read

http://www.geogebra.org/en/wiki/index.php/Release_Notes_GeoGebra_4.0) which include 8 new tools on the toolbar, handling of implicit curves, multivariate functions and inequalities, and more than 100 new features. Primary schools are now targeted by allowing an easier user interface for young users, and also professional users can benefit of the scripting technology and hundreds of new mathematical commands.

GeoGebra version 5.0 is also a closer milestone than it has ever been. This version extends GeoGebra with 3D features. The development is lead by Mathieu Blossier from University of Rouen, France. The talk will also show some demonstrations of this future version.

GeoGebra Institutes, Conferenes and News

Judit Pacsirta,
International GeoGebra Institute, Budapest

The establishment of GeoGebra Institutes were initially encouraged to be able to better support local communities in the use of GeoGebra. The three main aims of local institutes were offering training, development and conduct research within their community. Originally, we envisioned one GeoGebra institute per country, but later as we gained experiences it became clear that one institute is unable to serve its goals in most countries. Therefore, now we are changing the structure of GeoGebra Institutes and rather than establishing a single institute within a country we encourage forming GeoGebra Institute networks. In addition to the rapidly growing number of GeoGebra Institutes there will be numerous GeoGebra-related events and conferences in the new year. In my talk, I will explain the new ideas for the GeoGebra Institute Network and outline the challenges of



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event organisation. Furthermore, I will highlight the importance of getting GeoGebra known beyond the mathematics community.

support@geogebra.org

Using Simple Video Tutorials in Teaching with Geogebra

Matija Lokar

Faculty of Mathematics and Physics, University of Ljubljana

At the Faculty of Mathematics and Physics at University of Ljubljana students use many computer based mathematical tools. In a special course entitled Computer tools in Mathematics various tools and the possibilities of their application in practical problem solving are studied. A part of the resources available to students are the e-resources in the form of interactive tutorials. They explain basic features of tools used as well as they demonstrate solving different mathematical problems. The talk presents an approach towards creating such tutorials where GeoGebra is used. The software used in developing the tutorials is a freeware program Debug Mode Wink. These tutorials are produced by capturing screen snapshots during usage of GeoGebra. On each screenshot explanation boxes, audio, titles, navigation buttons and more can be added. The usage of navigation buttons allows the user of the tutorial to follow the construction easier and to adjust the speed of the presentation to match his level of understanding. The e-resources created for this course are also used in the project NAUK (<http://www.nauk.si>), where free e-resources from mathematics, physics, logic and computer science can be found and also at EU COMENIUS project European Development for the Use of Mathematics Technology in Classrooms (EduMatics).

Keywords: e-resources, video tutorials, GeoGebra

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Multimedia Approach in Teaching Mathematics Example of Lesson about the Definite Integral Application for Determining an Area

Marina Milovanović
University UNION Belgrade

: This paper presents the importance of using multimedia in the math classes by example of multimedia lesson about definite integral and the results of the research carried out among the students of the first years of faculty, divided into two groups of twenty five. One group had the traditional lecture about the definite integral while the other one had the multimedia method. The main information source in multimedia lectures was software created in Macromedia Flash, with definitions, theorems, examples, tasks as well as in traditional lectures but with emphasized visualisation possibilities, animations, illustrations etc. Both groups were tested after the lectures.



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Students from the multimedia group showed higher level of theoretical, practical and visual knowledge. Besides that, survey carried out at the end of this research clearly showed that students from multimedia group were highly interested in this way of learning.

Key words: multimedia learning, multimedia lessons, definite integral, area.

A New Teacher's Role and e-resources

Matija Lokar

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As stated in numerous papers and books the role of the teacher in the 21st century needs is redefined. They are no longer “walking encyclopedias” or “talking textbooks”. Instead, teachers are planners, strategists, researchers, pedagogical diagnosticians, work organizers, counselors, tutors, etc. Also a need on individual approach towards each student is more and more emphasized. It is very important that the teacher makes a proper combination of available teaching resources, suitable for the particular pedagogical situation. We should also not neglect the personal characteristics of each teacher, namely the way of teaching, personal beliefs, opinions on suitable motivational approaches etc. Therefore e-resources should be designed to be adaptable to the pedagogical situation and to the user, be it a learner or a teacher. In the talk we will describe several approaches how within NAUK group (<http://www.nauk.si>) e-resources are designed in an easily adaptable manner.

Keywords: e-resources, new teacher role, adaptation of resources

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Detecting patterns and properties in Geogebra constructions

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Dynamic geometry software, and Geogebra in particular, are effective tools for carrying out geometric constructions. Dynamic geometric constructions offer new ways for gaining conceptual understanding and for presenting proofs. Dynamic geometry software also enables effective checking of properties (e.g. whether three points in a configuration are collinear), but, apparently, there is no simple way for detecting properties. For example, in a dynamic construction it may occur that the length of a line segment is always equal to the sum of certain two other segments, or that certain four points lie on a same circle - but may not notice this at all.

We shall present a program, OK Geometry, that finds out common geometric properties in a dynamic geometric construction made, for example, with Geogebra.

We shall show that even simple constructions possess a rich variety of sometimes unexpected geometric properties. In a Geogebra construction OK Geometry detects various properties, ranging



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from metric properties of objects (e.g. congruent segments, angles, triangles) to similarity properties and projective properties (e.g. cross-ratio or points on a conic).

The detection of properties in a geometric construction is important for several reasons. Being aware of the properties is essential when solving certain geometric tasks, like advanced constructions, when constructing proofs, for generating hypotheses, even for generating exercises.

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Korištenje GeoGebre u nastavi matematike u Hrvatskoj

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Gimnazija i strukovna škola Jurja Dobrile, Pazin

Normala – udruga za promicanje nastave matematike, Zagreb

Na uporabu informacijsko komunikacijske tehnologije u nastavi matematike u Hrvatskoj posebnu je ulogu odigrala internetska zajednica nastavnika matematike okupljenih oko *mailing liste* smještene na javnom servisu smještene na javnom servisu *Yahoo Groups* (<http://groups.yahoo.com/group/nastava-matematike>). Grupa je pokrenuta 2000. godine. Osim općih tema vezanih uz struku posebno smo bili okrenuti potrazi za specijaliziranim računalnim programima za nastavu. Bili smo svjesni da otkako su daleke 1985. Judah Schwartz i Michal Yerushalmy napravili računalni program *The Geometric Supposer* za legendarni Apple II učenicima je otvoren put za eksperimentiranje u geometriji i dolaženja do vlastitih (makar već poznatih) otkrića. Nakon toga su nastali mnogi slični programi kao na primjer *The Geometer's Sketchpad*, *Cabri Geometry*, *Cinderella*, *Euklides*, *Winggeom*, *DrGeo*, *Geonext* i drugi. Programi dinamične geometrije su u protekla dva desetljeća obilježili svjetske konferencije nastavnika matematike.

Međutim, komercijalni programi zbog svoje cijene su predstavljali ozbiljan prepreku za njihovo korištenje u našim uvjetima. Tek s pojavom *GeoGebre* imali smo dovoljno kvalitetan program za uporabu u učionici. Godine 2004. krenuli smo u lokalizaciju *GeoGebre* 2.4 u trenutku kada je bila prevedena samo na četiri jezika. Iako je tada *GeoGebra* bila skroman program prednjačila je u produkciji potpuno funkcionalnih *Java* apleta, koji su se mogli objavljivati na webu ili bez prepreka međusobno dijeliti. Uz to, s pojavom *GeoGebre* više nije bila u pitanju samo manipulacija geometrijskim objektima, nego se na interaktivan i dinamičan način moglo pristupiti drugim matematičkim područjima. Širom su otvorene vrata primjeni informacijsko komunikacijske tehnologije s naglaskom na problemskom pristupu u nastavi matematike.

Novije verzije *GeoGebre* omogućile su interaktivnost, ne samo između korisnika i matematičkih objekata, nego i između konstrukcije (apleta) i teksta namijenjenog učeniku. To je bio poseban izazov za određen broj ljudi s kreativnim potencijalom unutar naše web-grupe da se ulovili u koštac s izradom digitalnih obrazovnih materijala. Prije svega riječ je o velikom broju apleta koji obuhvaćaju jednu temu, tvrdnju ili pojam, a po svojoj su naravi demonstracijski, objašnjavajući, heuristički, zabavni ili jednostavno slučajno generirani zadaci. S druge strane tu je i nekoliko nastavnih cjelina obrađenih kao programirani nastavni materijal, koji smo isprobali u više učionica. *GeoGebra* je odličan prezentacijski alat pa su neki radovi namijenjeni za prezentaciju uz LCD projektor u razredu. Na posljetku s *GeoGebrom* se može izraditi izvrsne slike za ispis ili za objavu na Internetu. Zahvaljujući Internetu, stručnim skupovima i metodičkom časopisu *Matematika i*



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škola ti materijali i sama *GeoGebra* dospjeli su do mnogih učionica u Hrvatskoj. Dio tog materijala prezentirat ću u izlaganju na Skupu.

Linkovi:

- Riznica apleta <http://apleti.normala.hr>
- Interaktivna matematika www.normala.hr/interaktivna_matematika
- GeoGebra Wiki <http://www.geogebra.org/en/upload> mapa *hrvatski*
- Matematika i škola <http://mis.element.hr>

Croatian experience in using GeoGebra

Josip Kličinović

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How should we teach and learn mathematics nowadays? How to use technology in teaching mathematics? Does the use of computers really change math class? Members of NGO Normala are actively seeking for answer on those questions. NGO deals with the promotion of teaching, improving teaching methods and resources, and applying modern technologies in teaching mathematics, cares for professional development and protecting the rights of teachers of mathematics.

We started as informal group of individuals gathered around a mailing list “Nastava matematike“ where we have collaborated on various projects and innovations in many years. In 2007. we started our NGO. We initiated many projects, localized GeoGebra on Croatian, organized GeoGebra workshops and courses for teachers and students. At first it was online course using Moodle, now we organize workshops in schools and on the obligatory professional development of teachers.

Our two biggest projects are “The Treasury of mathematic applets” and “Interactive mathematics” that we put on our website. “The Treasury” covers many concepts of elementary and secondary mathematics. “Interactive mathematics” is educational material that covers many units from the regular program in elementary and secondary (high) schools. Materials are fully dynamic and interactive, and shaped in a way that students can learn alone at home or in school with teacher assistance.

In this presentation I will give you an insight on our work and our experience in using GeoGebra in everyday work.





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Izometrijske transformacije u GeoGebra DGS

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U nastavi geometrije (kao u osnovnoj tako i u srednjoj školi) važnu ulogu imaju izometrijske transformacije. Posmatrajući osnosimetrične figure naučimo pojam osne refleksije. U daljem radu rotaciju, translaciju i centralnu refleksiju prikažemo kao kompoziciju dve osne refleksije, u zavisnosti od osa, da li se seku, da li su paralelne ili su međusobno normalne. Primena izometrija u osnovnim i interesantnim problemima. Primenom dinamičkog geometrijskog sistema, kao GeoGebra, učenje geometrije nastaje igra sa figurama. Vizuelizacija na računaru pomaže u shvatanju pojmova pa u daljem radu dobije se ideja rešenja i dokaza.

Keywords: izometrije, vizuelizacija, primena DGS

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Plane Mirror in GeoGebra Laboratory

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The best description that can serve as an abstract and introduction to this article is a quote from Michelangelo: "A drawing is the source and soul of every image and the root of every science." Keep in mind that the drawing in GeoGebra is live and dynamic. We can easily move the objects, the mirrors, and/or the rays, and change the angles to satisfy or break the reflection laws and do calculations followed by the desired results. Different cases of reflection and image formation in plane mirror, for example two intersecting mirrors, are explored in this paper using a dynamic and interactive worksheet created by GeoGebra.

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Matematičko modeliranje i GeoGebra

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U radu se prikazuju matematički modeli urađeni uz pomoć programskog paketa *GeoGebra*. Primeri su prilagođeni nastavnom planu i programu za srednju školu, a obuhvataju različite oblasti matematike, kao što su: kvadratna funkcija, eksponencijalna funkcija, geometrijske transformacije podudarnosti.



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Epsilon-delta definicija neprekidne funkcije– metod ilustracije programskim paketom GeoGebra

Duška Pešić

Gimnazija „Jovan Jovanović Zmaj“, Novi Sad

U izlaganju se daje predlog detaljne analize neprekidnosti nekih funkcija u tačkama njihovog domena, načini određivanja vrednosti parametra δ za različite vrednosti parametra ε , kao i donošenje konačnih zaključaka o neprekidnosti analiziranih funkcija metodom ilustracije uz korišćenje programskog paketa GeoGebra.

Transformacije grafika funkcije

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Sa transformacijama grafika funkcije učenici se sreću nekoliko puta u toku srednjoškolskog školovanja. Primena *GeoGebra*-e u nastavnom procesu učenicima se olakšava ne samo savlađivanje ove nastavne jedinice, nego i povezivanje sa drugim nastavnim sadržajima kako iz matematike tako i iz drugih predmeta.

On the polynomial approximations

Zlatko Udovičić

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We are considering the problem of the best polynomial approximation in a certain Hilbert spaces. Therein, we made two GeoGebra tools: the first one is for construction the mean-square polynomial approximation in the space of integrable functions, while the second one is for construction the least squares polynomial approximation in the space of the functions given at the discrete set of points. The second tool is generalization of the already existing GeoGebra tool “Best fit line” (that tool reduces to “Best line fit” in the case when the degree of approximating polynomial is equal to one). This approximation is also interesting since, in the case when the degree of approximation polynomial is equal to the number of given points minus one, it becomes Lagrange interpolating polynomial.

Keywords: mean-square, least squares, polynomial approximation, best line fit, interpolating polynomial

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Using GeoGebra for teaching and learning probability

Nenad Radaković

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This paper explores the use of GeoGebra as a tool for supporting high school students' understanding of probability. The research presented is part of a broader research project on improving high school students' risk literacy an important skill as decisions involving risk are made in all aspects of life including health, finances and politics.

Decisions involving risk are not only common, but they are also critical for individual and societal health and well-being. Despite its importance, there is a considerable evidence that most people are unable to adequately interpret and communicate risk.

One critical element for improving risk literacy is understanding of probability, particularly conditional probability and Bayes' theorem. I will present how features of GeoGebra could be used to support student learning, as well as shed understanding on their probabilistic thinking and reasoning.

Keywords: mathematics education, probability teaching and learning technology

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Demonstrative Approach to Ancient Constructive Problems through GeoGebra

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Some ideas to introduce ancient constructive problems to middle and high school students are shared herein. Free dynamic geometry software package GeoGebra is used to demonstrate approximate angle trisection, doubling a cube, and squaring a circle at a level that is accessible to the target audience. Such an approach enriches students' tools for heuristic search and exploration in problem solving.

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Geogebra - a Very Effective Tool for Teaching Mathematical Concepts and Properties

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¹University of Elbasan, ²University of Gjovik, Norway

In this paper we present some examples in Geogebra: visualization of the first derivative, monotonicity, extremums, properties of the function (strictly increasing/decreasing, and the Mean Value Theorem. The results and the conclusions are based on the experiment carried out in the teaching process in the chapter of derivatives in a third year class of a secondary school in Albania. Also, there are some encouraging facts obtained by the use of GeoGebra. Namely, the double representation and the dynamic feature of GeoGebra allows the students to quickly grasp the mathematical concepts and properties and then be actively involved in further explorations. Using GeoGebra, it is easier for the teachers to explain mathematical concepts, in particular the properties of algebraic objects.

Keywords: Dynamic demonstration, Visualization

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Mathematical games can encourage pupils to learn and use the software package “GeoGebra”

Ivana Sladoje

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In this paper we propose realization of a number of informatics lessons in higher grades of primary school with the aim to encourage the development of key competencies in the area of acquiring IT knowledge and skills, as well as to contribute to the development of social-emotional competencies of pupils.

The suggested activity of pupils is to play mathematical games supported by a computer, while being organized in groups and participating in a competition. For this purpose it is necessary for pupils to first explore and learn to use the software package "GeoGebra", being an excellent tool for displaying elements and moves when playing the suggested games. In this way we achieve the constructivist approach to teaching: learning is building of knowledge, because it connects knowledge from various fields. At the beginning, mathematical games provide a tool and motivation for acquiring IT knowledge, whereas later on acquired computer knowledge enables that the software package "GeoGebra" becomes a teaching tool for acquiring new knowledge in mathematics. Necessary motivation comes from the children's natural affinity to play. Their awareness of the existence of a winning strategy in the proposed mathematical game makes the challenge even bigger. Cooperative teaching intensifies the collaborative peer interaction that additionally contributes to the social learning.



The paper presents the results of a study exploring the affinities of almost 600 pupils in upper grades of primary school. The focus was on pupils' opinions regarding teamwork, their preferences regarding forms of teaching, their interests in solving mathematical problems, as well as in team competitions, and their habits regarding the use of computers for playing games. In addition to scientifically proven positive aspects of the forms of teaching suggested in the proposed organization of classes, reported opinions and preferences of pupils are the key factors which motivated us to design and organize the proposed computer science lessons.

Znak i monotonost funkcije

Dragana Nedić

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Ispitivanje znaka, a i monotonosti funkcije predstavlja problem učenicima, počev od kvadratne funkcije pa dalje. GeoGebra je matematički paket koji nam omogućava da učenicima približimo te pojmove, tako da oni na jednostavan i zanimljiv način dosta lako nauče da sa grafika funkcije prepoznaju i pročitaju intervale na kojima je funkcija pozitivna, odnosno negativna, intervale gdje funkcija raste, odnosno opada.

Geogebra i matematičko modelovanje u nastavi matematike

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Departman za matematiku i informatiku, Novi Sad

Na primeru matematičkog modela koji je u visokoškolskoj nastavi matematike korišćen za realizaciju nastavne jedinice na temu: *Izvod funkcije i njegove primene*, razmatra se i uloga drugih alata, kao što je GeoGebra.

Prikazani su različiti načini ilustracije pojma izvoda funkcije koji su realizovani u GeoGebra-i i razmatra se uticaj primene metode matematičkog modelovanja, poboljšane sa primerima izvedenim u GeoGebra-i, na kvalitet stečenih znanja studenata vezanih za pojam izvoda funkcije i njegove primene.

Takođe se analizira i efekat kombinovane primene GeoGebra-e i metode matematičkog modelovanja u visokoškolskoj nastavi matematike, posmatran iz ugla didaktike matematike.

Ključne reči: GeoGebra, matematičko modelovanje, nastava matematike, izvod funkcije

Exploring and discovering mathematics with dynamic worksheets

Dessislava Dimkova

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New information technologies offer new tools in teaching, particularly in mathematics. Currently, we use a variety of dynamic software systems, however, not always effectively, unless a developed methodology is used. Some teachers use this software only in order to avoid drawing on the board, others use it to move objects on the screen – however, students are just observers. Opponents say that the new technologies only distract students and stick to the traditional teaching methods.

Here, we offer a classic lesson integrated with a dynamic software. A convenient way to implement this idea is to use a browser. The possibility of combining a dynamic construction, text, graphic information and links converts students into researchers and designers, in problem-solvers. All this takes place in a unified environment – in a dynamic worksheet. Using one of the available dynamic software systems - GeoGebra, relying on traditional teaching methods (i.e., we don't abandon theorems along with their respective proofs and corollaries), we'd like to encourage students to explore, examine, prove... In other words, we'd like to provide them with objects to think with.

To illustrate these ideas, we present a cycle of lectures on geometric transformations, demonstrating our main educational principle: "Classic mathematics + new technologies = opportunity for investigations, constructions, explorations, hypotheses, proofs. This principle has been implemented in a user-friendly platform providing opportunities for the students to solve mathematical problems and create projects reflecting their personal interests and potential.

Vector-Valued Functions and Geogebra

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Calculus courses at the universities we represent have included vector-valued functions and motion in plane and in space. Using Geogebra one can facilitate learning such concepts. Sketching the graph of real valued function $y = f(x)$ in GeoGebra is extremely easy. In this paper, we consider a way of using GeoGebra for sketching the graphs of 2D vector functions and needs for 3D approach. For parameterized 2D curve, GeoGebra slider can be used to determine 2 coordinates of the point. Setting the "trace on" option to the point, and also "animation on" option to the slider, one can obtain effective way for explaining the particle's motion in the plane. It is not difficult to show the tangent vector to the curve, as effective way for visualization the meaning of the derivatives of the vector-valued function as a particle's velocity vector. We are going to show this approach using examples like astroid, cardioid and other 2D curves as well.

After all, it is clear why needed 3D approach for teaching vector valued functions is. We tried to use GeoGebra 3D Beta to create instructional materials according to 3D parameterized curves, but it is not powerful as for example using Mathematica or Matlab software we usually use for that purpose.





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Just to explain needed visualization power in 3D, we are going to show several animations created in Matlab.

Keywords: Vector valued functions; parameterized 2D curve; parameterized 3D curve; derivatives; tangent vectors; velocity vector.

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Learning Trigonometric Functions Using GeoGebra

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Students in middle school education during grades 9-12 are introduced to trigonometric functions. First they understand these functions as simple ratios of side lengths with respect to an acute angle. Then, they study the characteristics of these functions whose argument is any angle and finally they study sinusoidal functions. In this article, we look at using GeoGebra to give continuity to this process and to improve understanding.

One of the steps in learning trigonometric functions of an arbitrary angle is to use the unit circle. With GeoGebra, the teacher or student creates a unit circle at $(0,0)$ and an angle slider to create a dynamic, movable point on this circle. He uses the coordinates of this point to draw the trigonometric functions directly. This relates his previous understanding and at the same time helps him to understand the changing signs, monotonicity, domain, range and boundedness of these functions.

With the same worksheet we use the arc length to the movable point to understand radian measure and its relation to the angle, circle and the trigonometric functions.

Finally, we use ready-to-use interactive GeoGebra worksheets to study and understand the sinusoidal functions. These interactives have sliders for all of the relevant constants a , b , c and d so that this exploration is both algebraic and geometric-visual.

We used a video camera to record this classroom process and there is a short presentation of this. When we finished our lessons with the help of GeoGebra, we surveyed our students to see the results of using this methodology for learning trigonometric functions and the results of this survey will be presented in this paper.

Learning Trigonometric Functions Using GeoGebra

Zoran Trifunov
Univerzitet Sv. Kliment Ohridski -Bitola

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Keywords: trigonometric functions, unit circle at $(0,0)$, movable point on this circle
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Computer Algebra Systems Supporting Teaching/ Learning Linear Algebra

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Most of the faculties for engineering and informatics at the universities implement the computer algebra systems Mathematica or Matlab during the contemporary lab classes in mathematics. First year students at universities are usually not familiar with any of the CAS or DGS and show lack of computer supported mathematics. This paper presents some possibilities to help the upper secondary school students in overcoming this problem and prepare them for university mathematics into lab.anadonev@gmail.com

Problem Solving Using GeoGebra

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Problem solving is an important component of teaching mathematics. It is a way to present mathematics and it is also a skill which enhances logical reasoning. In this paper, two different approaches towards the problem solving will be presented. The first one uses classical method and the other one uses GeoGebra. The steps of both solutions of several math problems of different levels will be comparatively analyzed in order to shed light on the specific benefits of each. The



group of students from the third grade, who are familiar with GeoGebra, took part in a number of exercises and their experience in this specific problem solving will also be presented.

Problems of different levels and several mathematics topics (such as geometry, analytic geometry or algebra) have been considered in this work. The seventeen -year-olds chosen for this study are those who are a bit more interested in mathematics than the others. At the time when they were doing these exercises, they were studying in the third grade (out of four) of mechanical engineering high (secondary) school in Novi Sad.

Keywords: Math teaching, problem solving, logical reasoning, GeoGebra
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Razvoj softvera za dinamičku geometriju na Silverlight platformi

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Danas postoji mnogo različitih programa za dinamičku geometriju, od kojih su neki besplatni, a neki ne. Ukoliko želimo da stvorimo softverski sistem koji u sebi sadrži već postojeći softver za dinamičku geometriju, sa jedne strane smo ograničeni na upotrebu besplatnog softvera, a sa druge strane moramo paziti na međusobnu kompatibilnost korišćenih softverskih platformi. Poseban problem predstavlja to što nijedan od postojećih softvera nije ispunjavao sve naše zahteve. Iz tog razloga smo počeli da razvijamo sopstveni softver za dinamičku geometriju, sastavljen od komponenti, koje se mogu koristiti zajedno, ali i zasebno, kao samostojeći proizvod ili kao deo nekog većeg softverskog sistema. Naše izlaganje sadrži prikaz do sada postignutih rezultata, kao i pravce daljeg razvoja.

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Development of ideas in a GeoGebra – aided mathematics instruction

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With GeoGebra introduced in mathematics instruction the teaching/learning process is not improved in terms of speed and quality only. Mathematical concepts, rules and procedures must be adjusted to the new environment. The paper deals with development of ideas when the concept of ellipse is being built with GeoGebra. We shift from dynamic applets that show the ellipse characteristics to applets that are built by students and teacher together. The process of building new applets enables students discover independently the ellipse characteristics. In other words, we use GeoGebra to move from mechanical to creative way of thinking.

Keywords: ellipse, dynamic applets, creative thinking

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Using GeoGebra and Modern Web Technologies for Teaching Analytic Geometry

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This talk will present ways of combining dynamic mathematical package GeoGebra with modern Web technologies (markup languages XHTML and MathML) within high school education. These technologies were used to create a small electronic textbook in analytic geometry. Some examples from the textbook will be presented in the talk. Inspiration for making this textbook is the need for visualisation and representing mathematical content on Internet and increasing pupils motivation through using interactive Web pages and applets.

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Kompleksni brojevi u programu *Geogebra*

Miljana Artonović,

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Samo uvođenje pojma kompleksnog broja, u trećoj godini srednje škole predstavlja problem učenicima, počev od definicije i osnovnih osobina jednačine: $x^2 + 1 = 0, x \in R$, u kojoj nema rešenja u skupu realnih brojeva R .

Matematički paket GeoGebra nam omogućava da učenicima i grafički i algebarski prikazemo pojmove kompleksnog broja. Počevši od: (0,0)-kompleksne nule; (1,0)- kompleksne jedinice; i (0,1)- imaginarne jedinice; do trigonometričkog oblika kompleksnog broja. Prednost ovog matematičkog paketa GeoGebra je sam kreativniji pristup metodike nastave matematike, koji učenicima približava predmet na što zanimljiviji i pristupačniji način.

Pored profesora, ovaj program i učenici mogu na jednostavniji, zanimljiviji i konkretniji način koristiti pri rešavanju matematičkih problema.

Kinematika kosog hica

Aleksandar Bukva, Raičević Zivko,

učenici Gimnazije "Jovan Jovanovic Zmaj"

U radu se daje teorijski uvod u kosi hitac, a zatim se pomoću programskog paketa *GeoGebra-e* vizualno prikazuju odgovarajući primeri iz matematike i fizike, vezani za kosi hitac, sa posebnim osvrtom na simulaciju šutiranja lopte.



Utvrđivanje reda hemijske reakcije

Milanović Ivana, profesor,
Raičević Vidak, učenik trećeg razreda,
gimnazija "Isidora Sekulić" Novi Sad

U radu je izložen primer korelacije matematike i hemije, odnosno hemijske mehanike u nastavi. Brzina hemijskih reakcija u zavisnosti od koncentracije reaktanata ovde je opisana matematičkim jednačinama, koristeći diferencijalni i integralni račun. Zatim se pristupilo rešavanju problema određivanja reda hemijske reakcije, a koji je u vezi sa brzinom reakcije. Problem je rešen primenom matematičkog modela, u okviru koga su se koristila svojstva linearne funkcije.

U radu je korišćen programski paket GeoGebra, posebno upotreba tabela, formiranje liste tačaka, njihovo grafičko predstavljanje, fitovanje krivih.

Ključne reči : međupredmetna korelacija, matematički model, brzina i red hemijske reakcije, izvod, integral, linearna funkcija, fitovanje

Establishing the Order of Chemical Reaction Using a Mathematical Model

Milanović Ivana, teacher,
Raičević Vidak, student,
gimnazija "Isidora Sekulić" Novi Sad

The paper presents an example of correlation between Mathematics and Chemistry or to be more precise, chemical mechanics within teaching. The speed of chemical reactions is described with the help of mathematical equations, using differential and integral calculus and it depends on the concentration of the reactants. Determining the order of chemical reaction follows, which correlates with the speed of reaction. The problem is solved by mathematical models, within which properties of linear functions were used.

The paper uses a software package GeoGebra, especially the use of charts, forming a list of points, their graphical representation, fitting curves.

Key words: inter-subject correlations, mathematical model, speed and order of chemical reaction, derivation, integral, linear function, fitting

Testiranje i provera znanja na Moodle sistemu

Marina Petrović
Pedagoški fakultet, Sombor

Testovi na Moodle sistemu sadrže vrste zadataka otvorenog i zatvorenog tipa i veliki broj mogućnosti za kreiranje, upravljanje, ocenjivanje i analizu rezultata testova. U radu posvećujemo pažnju testovima. Prikazaćemo izgled i postupak kreiranja svakog tipa pitanja u okviru banke





*Jó szomszédok a közös jövőért
Good neighbours creating common future
Dobri susedi zajedno stvaraju budućnost*

University of Szeged - UNS Faculty of Science Novi Sad

Teaching Mathematics and Statistics in Sciences HU-SRB/0901/221/088

pitanja, odabir i postavljanje pitanja u konkretan test, podešavanja parametara testa, način izrade testa od strane učenika, pregled rezultata.

Na primeru jednog gotovog testa na temu eLearning 2.0. biće pokazan izgled mogućnosti analize koju nudi Moodle. Test je postavljen na sistem Moodle u okviru seminara za stručno usavršavanje nastavnika osnovnih i srednjih škola Srbije, pod nazivom "Alati za e-učionicu".

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Grafički prikaz trigonometrijskih funkcija

Danka Lučić

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Korišćenjem programskog paketa GeoGebra prikazaće se vizualizacija trigonometrijskih funkcija sa posebnim osvrtom na Furijeove redove.

Ekstremne vrednosti funkcije i primene

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Korišćenjem programskog paketa GeoGebra prikazaće se vizualizacija ekstremnih vrednosti funkcija sa posebnim osvrtom na zadatke vezane za probleme u geometriji.

Logarithmic function and GeoGebra

Tijana Stojančević

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In the talk we will present some possibilities of the programme package *GeoGebra* for the visualization of the properties of logarithmic function.

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