G2. Mathematical basics: functions, linear functions, power functions and applications

Function: A function assigns a unique element of a set Y, to an element of a set X. The elements of the sets X and Y can be of any type, but now we only consider the case when the elements of these sets are real numbers.

Domain: The set X, or a subset of X

Codomain: a subset of Y

Linear functions: A linear function takes the form $y=mx+b$.

Slope (m): when x increases by a unit, y changes by m units. If m is positive, y increases when x increases; if m is negative, y decreases when x increases.

Constant (b): the value, where the graph crosses the y-axis.

The equation of a line crossing a given point x_0, y_0 (in the cartesian plane), with given slope m:

$$y - y_0 = m(x - x_0).$$

The equation of a line crossing two given points (x_1, y_1) and (x_2, y_2): $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1).$

Power funtions: $y=x^\alpha$.

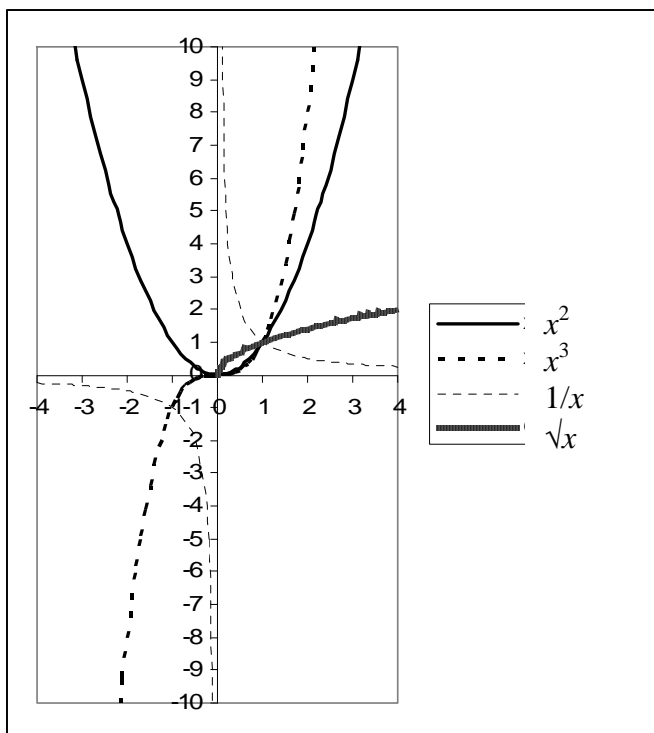Special cases:

$y=x^2$, sqaure function, parabola. Peak-point equation: $y - y_0 = a(x - x_0)^2$, where (x_0, y_0) denotes the coordinates of the peak-point, and a is an elongating factor ($|a|>1$), or a shrinkage factor ($0<|a|<1$) or a mirror to the x axis ($a=-1$).

$y=x^3$, cube function

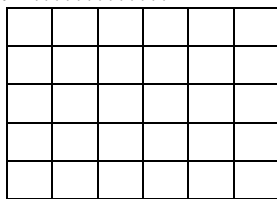$y=x^{-1}=1/x$ reciprocal function

$$y = x^{1/2} = \sqrt{x}$$

Exercises

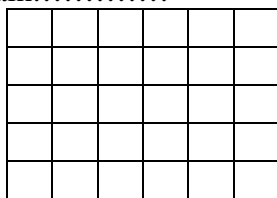2.1. Determine if the maps given below are functions (and draw a plot):

2.1.1. Is it a function?……………. Domain:………… Range:………..

x	y
1	1
2	1
3	3
4	4

2.1.2. Is it a function?……………. Domain:………… Range ………..

x	y
1	1
1	2
3	3
4	4

2.1.3 Is it a function?……………. Domain:………… Range ………..
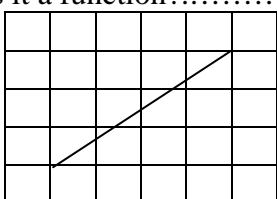
2.1.4 Is it a function?……………. Domain:………… Range ………..
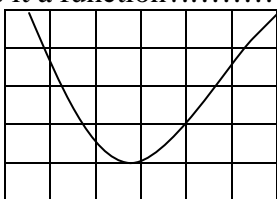
2.1.5 Is it a function?……………. Domain:………… Range ………..
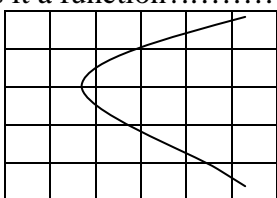
2.2. Linear functions.

2.2.1. Plot the following linear functions:
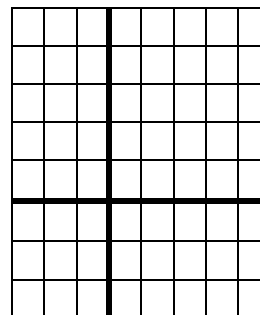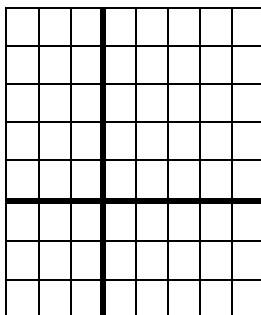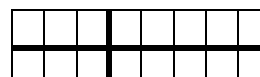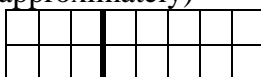
$y=2x-1$
$y=-2x+1$
$y=x$
$y=-x$
$y=x/2+1$
$y = \dfrac{2}{3}x$
$y = \dfrac{3}{2}x+1$

2.2.2. Give the equation of the linear functions (approximately)

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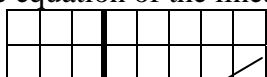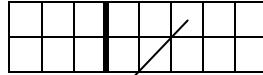
2.2.3. Answer the following questions, based on the plots!
Questions about the plots:

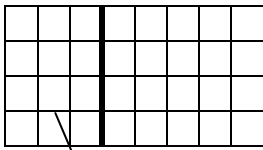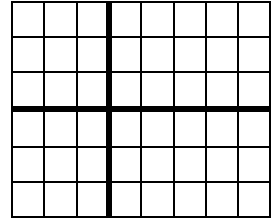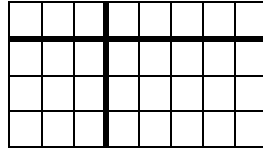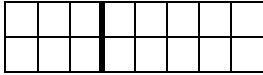|		
A	What's the independent variable (x)?	
	What's the dependent variable (y)?	
	Give the slope of the line!	
	Does the line increase or decrease? Why?	
	Give the equation of the line	
	What do you think, does the linear function fit to the data well?	
B	What's the independent variable (x)?	
	What's the dependent variable (y)?	
	Give the slope of the line!	
	Does the line increase or decrease? Why?	
	What do you think, does the linear function fit to the data well?	
C	What's the independent variable (x)?	
	What's the dependent variable (y)?	
	Give the slope of the line!	
	Does the line increase or decrease? Why?	
	Give the equation of the line	
	What do you think, does the linear function fit to the data well?	
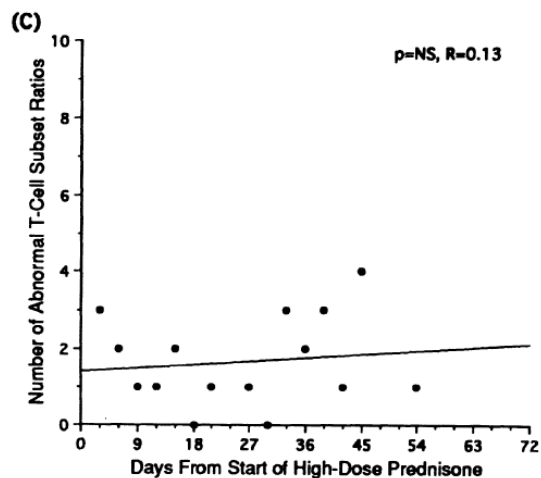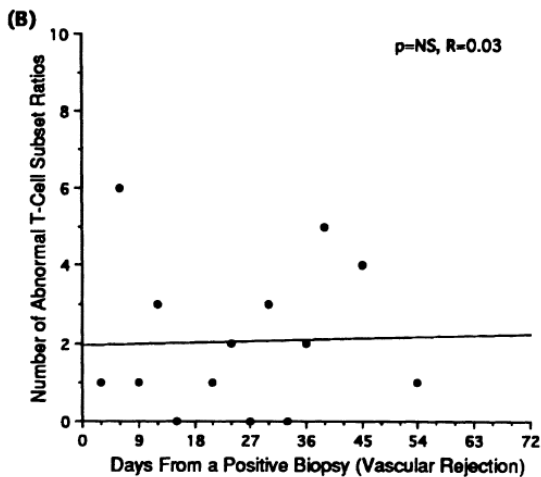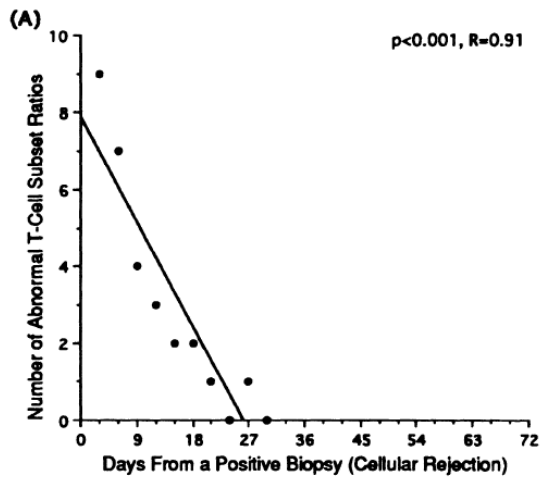
FIG 3. A, Plot of the number of abnormal subset ratios (in 3-day intervals) plotted as a function of time from a biopsy positive for cellular rejection. Regression analysis of this plot indicated a significant correlation ($P < .001$, $R = .91$) between these occurrences. B, Plot of the number of abnormal subset ratios plotted as a function of time from a biopsy positive for vascular rejection. Regression analysis of this plot indicated no correlation between these occurrences. C, Plot of the number of abnormal subset ratios plotted as a function of time from the start of high-dose prednisone immunosuppressive therapy. Regression analysis of this plot indicated no correlation between these occurrences.

JF Carlquist, ME Hammond, RL Yowell, C Holland, S Swanson and JL Anderson: Correlation between cellular rejection of cardiac allografts and quantitative changes among T-cell subsets identified by V beta epitope expression. Circulation 1994;90;686-693

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2.2.4. Find the slope of the linear funcion obtained by the regression! What do you think, does the linear function fit to the data well?
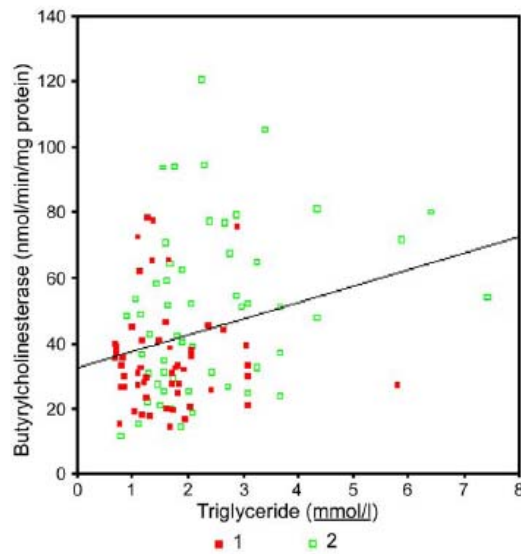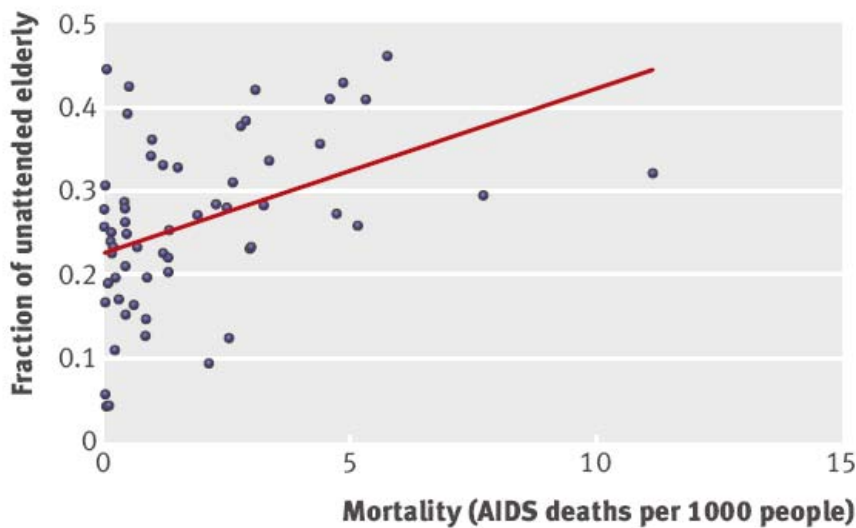
Fig. 2. Correlation of serum butyrylcholinesterase and triglyceride levels (1. Control, 2. Hyperlipidaemic group).


Fig 1| AIDS mortality rate and the proportion of older people living alone without any adults between the ages of 18 and 59. The line represents unadjusted linear regression showing higher fractions of unattended elderly people associated with higher AIDS mortality rates. The primary analysis uses a probit regression

Kautz, T. et al: AIDS and declining support for dependent elderly people in Africa: retrospective analysis using demographic and health surveys. BMJ 2010;340:c2841.
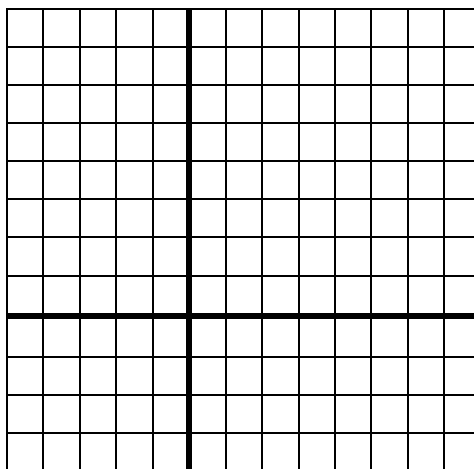
2.3. Power functions.

2.3.1. Plot the following functions!

$y=x^2 + 2$

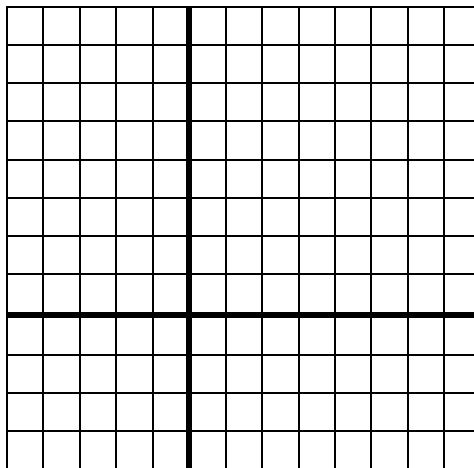$y=x^2 - 2$

$y=(x+2)^2$

$y=(x-2)^2$

$y=-x^2 + 2$

$y=(x-3)^2 + 2$
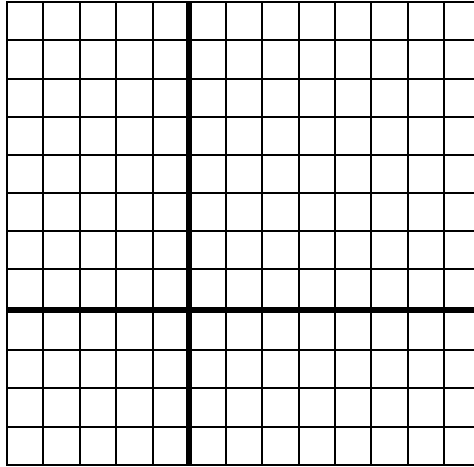
2.3.2. Plot the following functions!

$y=x^3 + 2$

$y=(x+2)^3$

$y=(x-2)^3$

$y=-x^3 + 2$

$y=(x-3)^3 + 2$

2.3.3. Plot the following functions!

$y=1/x + 2$

$y=1/x - 2$

$y=1/(x+2)$

$y=1/(x-2)$

$y=1/(x-2) + 3$

2.3.4. Expand!

$(x+2)^2 =$ $\qquad$ $(x-2y)^2 =$ $\qquad$ $[a-(b/2)]^2 =$

2.3.5. Plot the following power functions, and give their domains and ranges! Find the values assigned to –1, 0, 1 (if sensible).

a) $y = x^2$ b) $y = x^3$ c) $y = x^{-1}$ d) $y = x^{-2}$ e) $y = x^{-3}$ f) $y = x^{1/2}$

g) $y = -x^2$ h) $y = -x^3$ i) $y = -x^{1/2}$