Biostatistics

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www.model.u-szeged.hu
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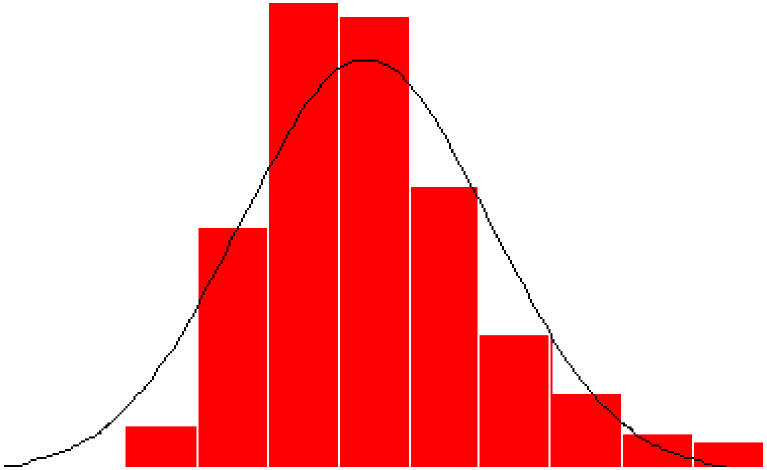
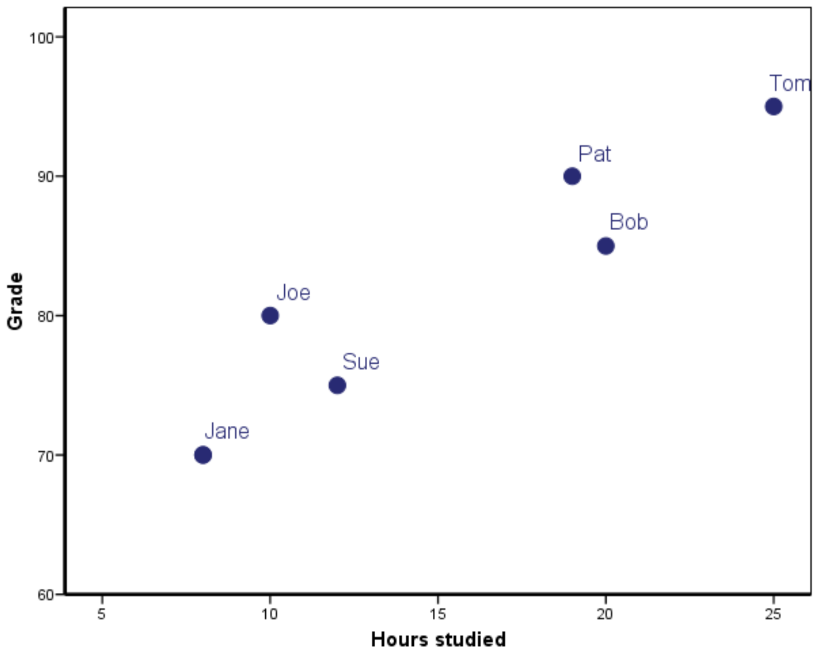
Correlation, linear regression



Scatterplot

Relationship between two continouous variables

Student	Hours studied	Grade
Jane	8	70
Joe	10	80
Sue	12	75
Pat	19	90
Bob	20	85
Tom	25	95



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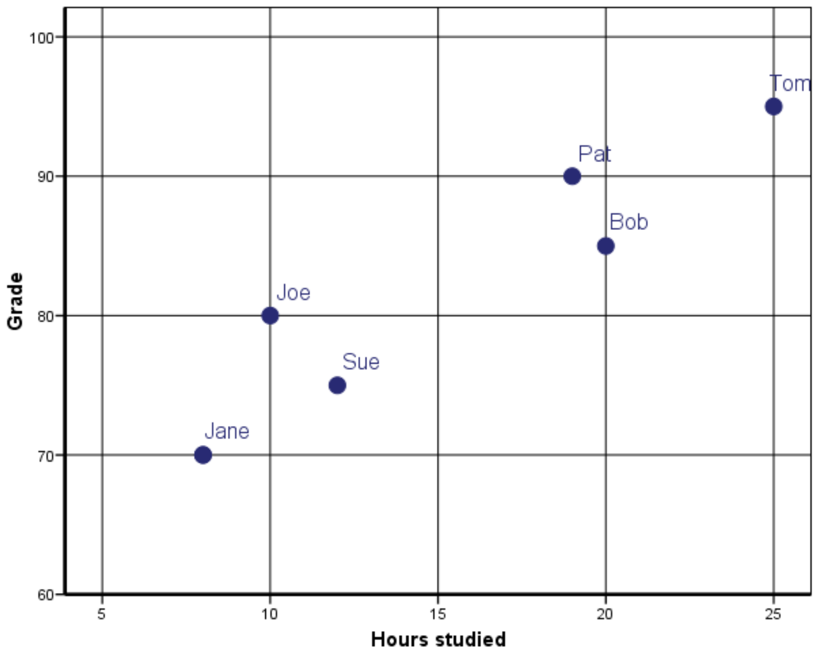
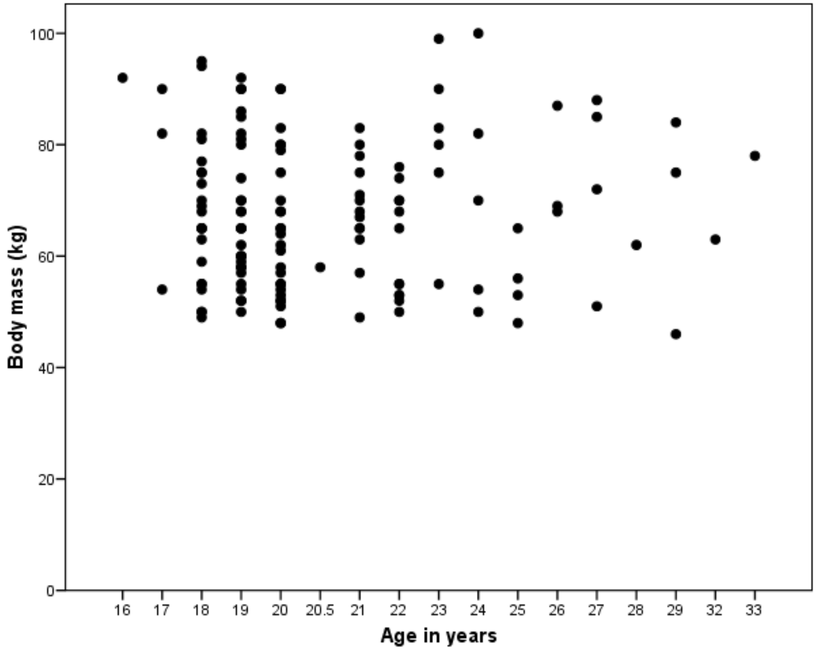
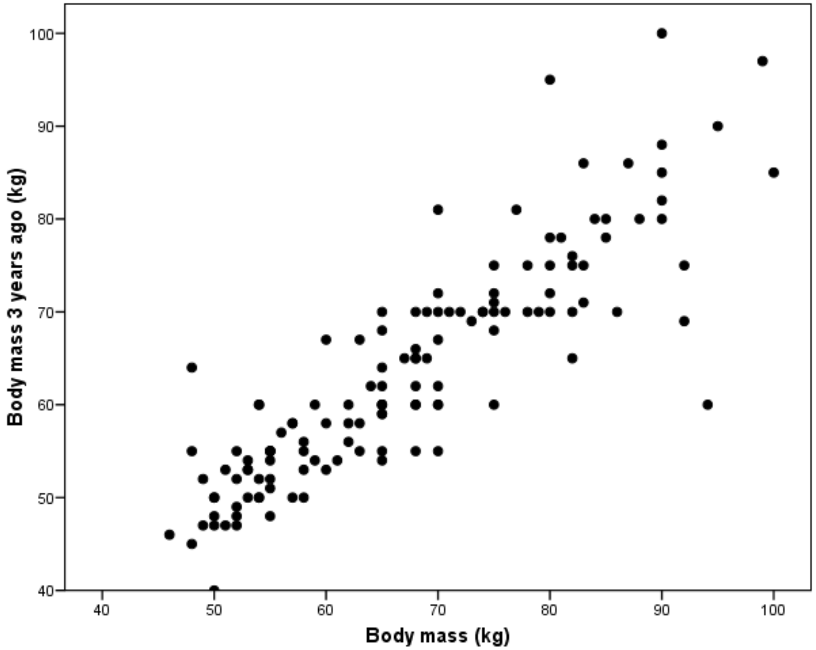


Scatterplot

Other examples





Example II.

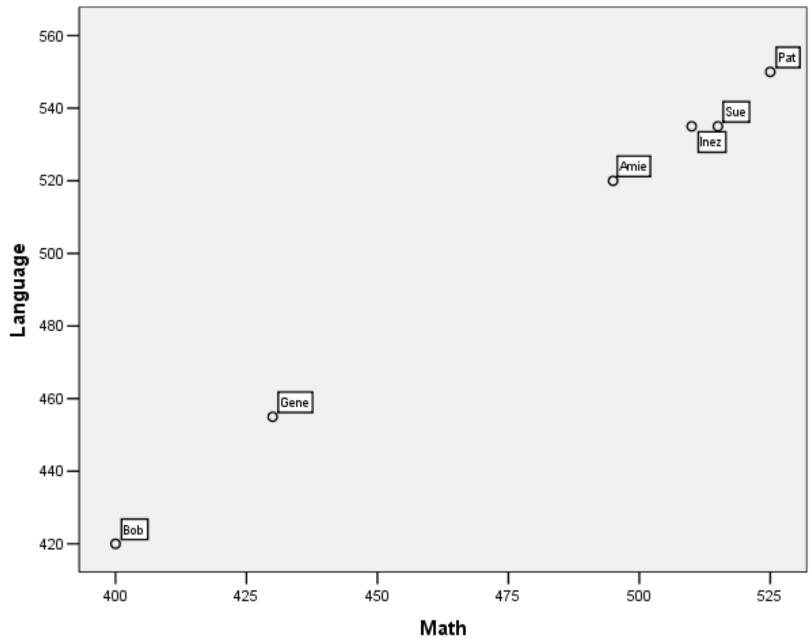
- Imagine that 6 students are given a battery of tests by a vocational guidance counsellor with the results shown in the following table:

	STUDENT	RETAIL	THEATER	MATH	LANGUAGE
1	Pat	51.00	30.00	525.00	550.00
2	Sue	55.00	60.00	515.00	535.00
3	Inez	58.00	90.00	510.00	535.00
4	Arnie	63.00	50.00	495.00	520.00
5	Gene	85.00	30.00	430.00	455.00
6	Bob	95.00	90.00	400.00	420.00

- Variables measured on the same individuals are often related to each other.

Let us draw a graph called scattergram to investigate relationships.

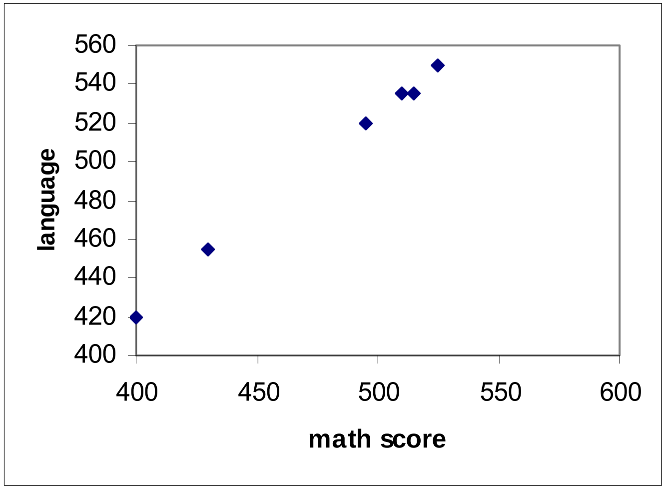
- Scatterplots show the relationship between two quantitative variables measured on the same cases.
- In a scatterplot, we look for the direction, form, and strength of the relationship between the variables. The simplest relationship is linear in form and reasonably strong.
- Scatterplots also reveal deviations from the overall pattern.



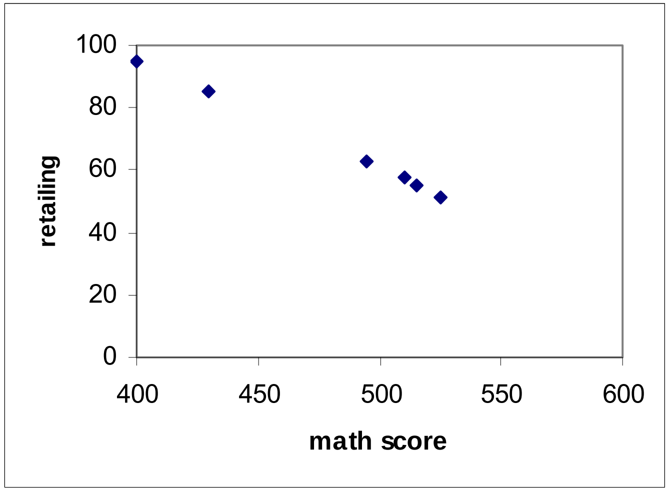
Creating a scatterplot

- When one variable in a scatterplot explains or predicts the other, place it on the x-axis.
- Place the variable that responds to the predictor on the y-axis.
- If neither variable explains or responds to the other, it does not matter which axes you assign them to.

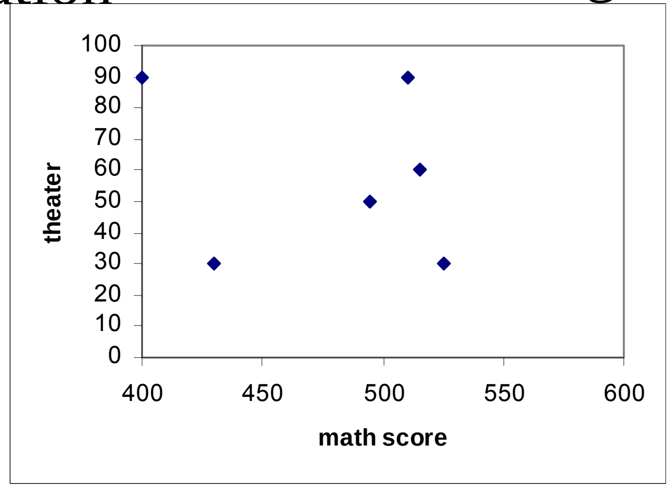
Possible relationships



positive correlation



negative correlation



no correlation

Describing <u>linear</u> relationship with number: the coefficient of correlation (r).

Also called **Pearson** coefficient of correlation

- Correlation is a numerical measure of the strength of a linear association.
- The formula for coefficient of correlation treats *x* and *y* identically. There is no distinction between explanatory and response variable.
- Let us denote the two samples by $x_1, x_2, \ldots x_n$ and $y_1, y_2, \ldots y_n$, the coefficient of correlation can be computed according to the following formula

$$r = \frac{n \cdot \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{\left(n \cdot \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2\right)\left(n \cdot \sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} y_i)^2\right)}} = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \overline{x})^2 \sum_{i=1}^{n} (y_i - \overline{y})^2}}$$

Karl Pearson

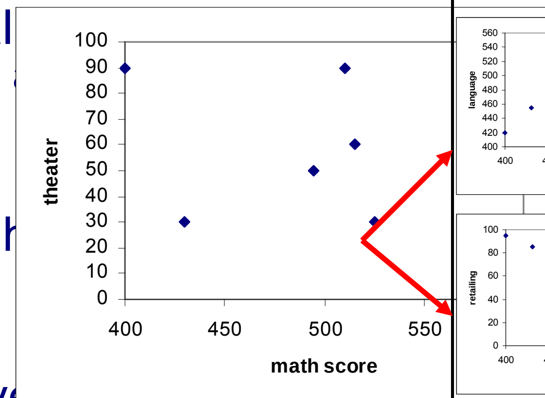
- Karl Pearson (27 March 1857 – 27 April 1936) established the discipline of mathematical statistics. http://en.wikipedia.org/wiki/Karl_Pearson

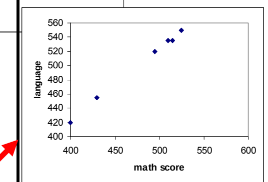
Properties of *r*

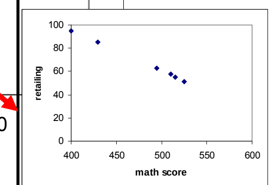
- Correlations are between -1 and +1; the val
between -1 and 1, either extreme indicates
association.

$$-1 \le r \le 1.$$

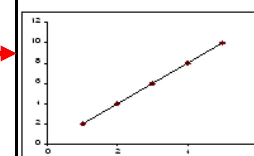
- a) If *r* is near +1 or -1 we say that we have h

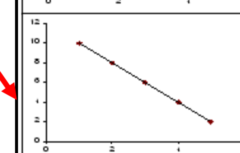






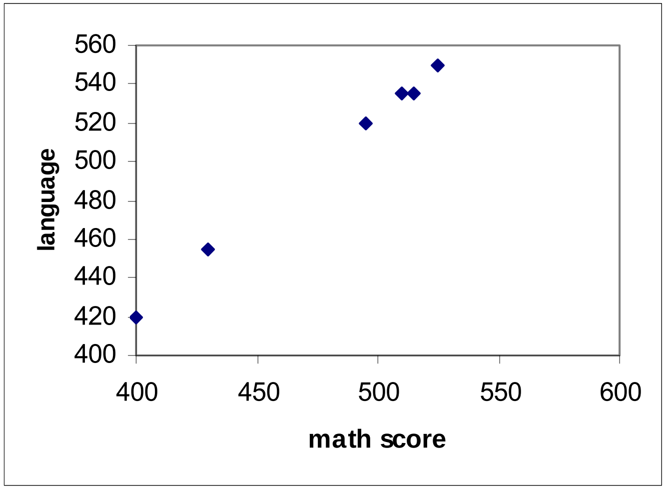
- b) If *r*=1, we say that there is perfect positive correlation.
If *r*= -1, then we say that there is a perfect negative
correlation.



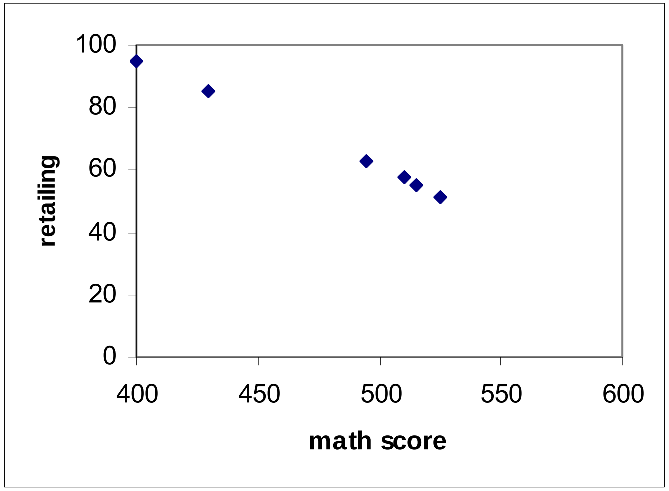


- c) A correlation of zero indicates the absence of linear association. When there is no tendency for the points to lie in a straight line, we say that there is no correlation (*r*=0) or we have low correlation (r is near 0).

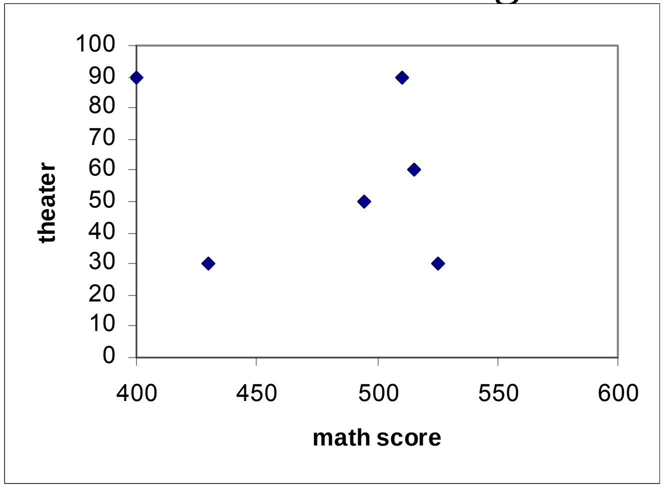
Calculated values of r



positive correlation, **r=0.9989**

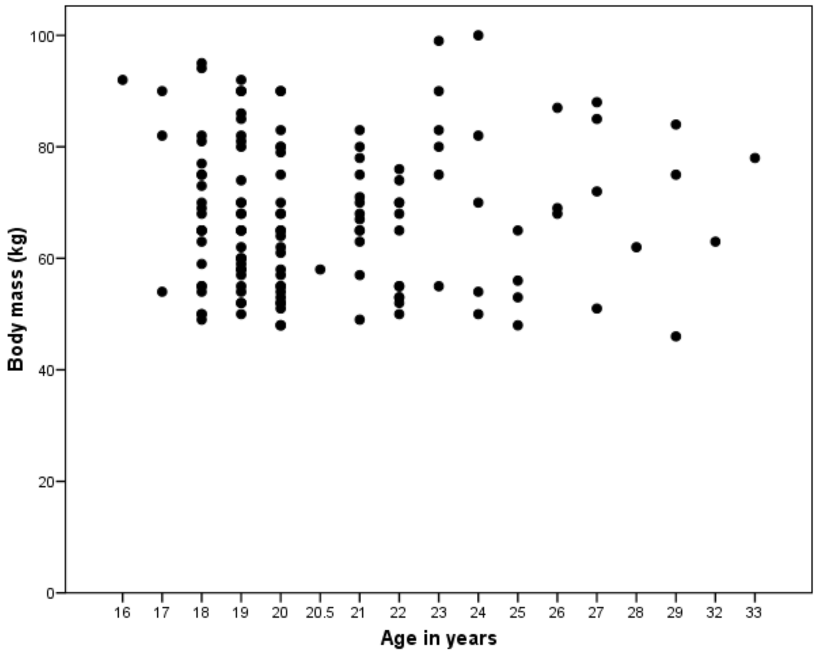


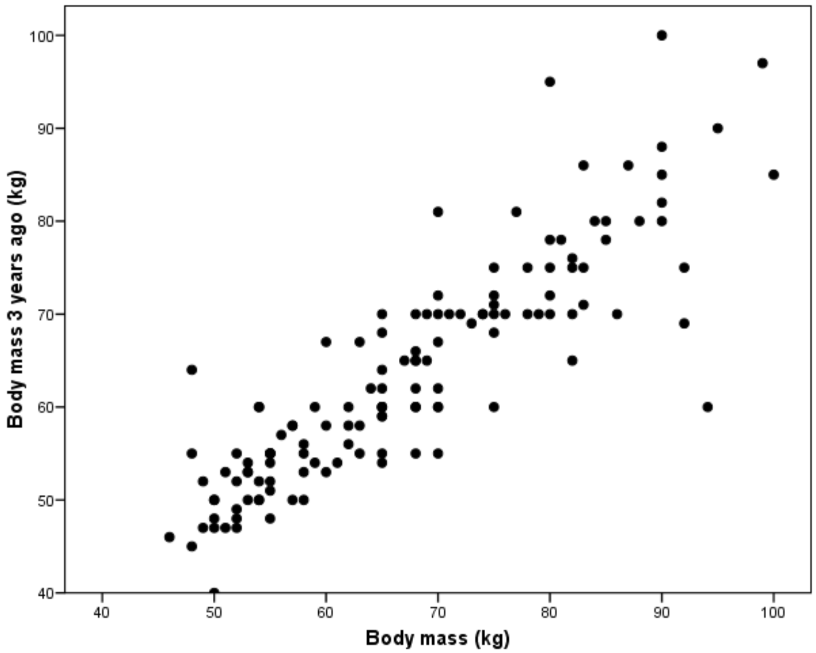
negative correlation, **r=-0.9993**



no correlation, **r=-0.2157**

Scatterplot
Other examples





r=0.018

r=0.873

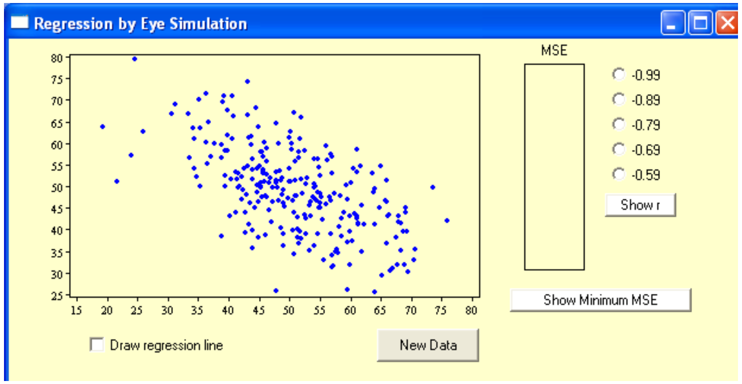
Correlation and causation

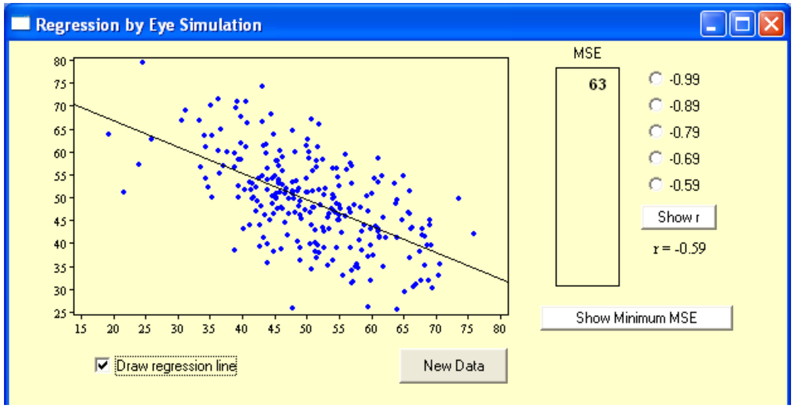
- a correlation between two variables does not show that one causes the other.

Correlation by eye

http://onlinestatbook.com/stat_sim/reg_by_eye/index.html

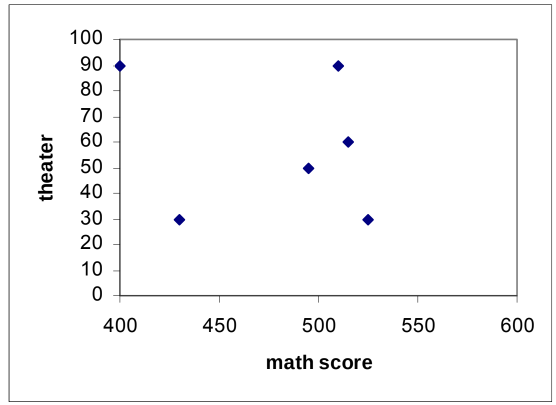
- This applet lets you estimate the regression line and to guess the value of Pearson's correlation.
- Five possible values of Pearson's correlation are listed. One of them is the correlation for the data displayed in the scatterplot. Guess which one it is. To see the correct value, click on the "Show r" button.



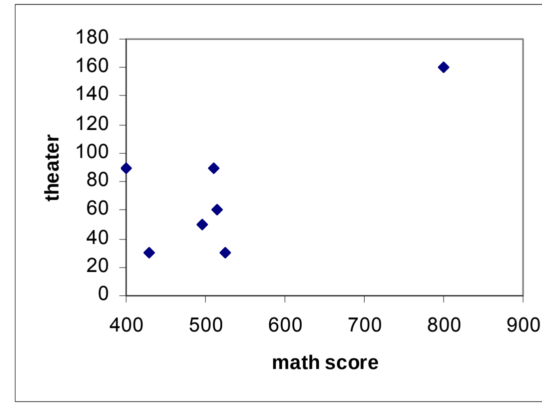


Effect of outliers

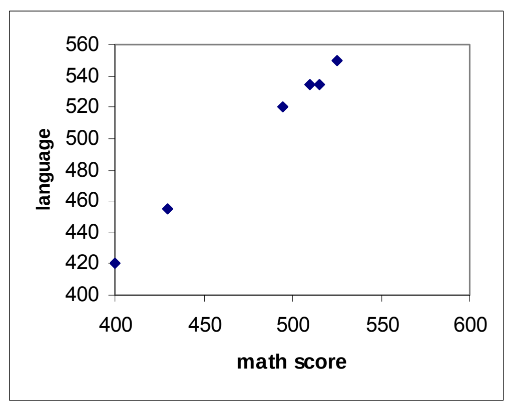
- Even a single outlier can change the correlation substantially.
- Outliers can create
  - an apparently strong correlation where none would be found otherwise,
  - or hide a strong correlation by making it appear to be weak.



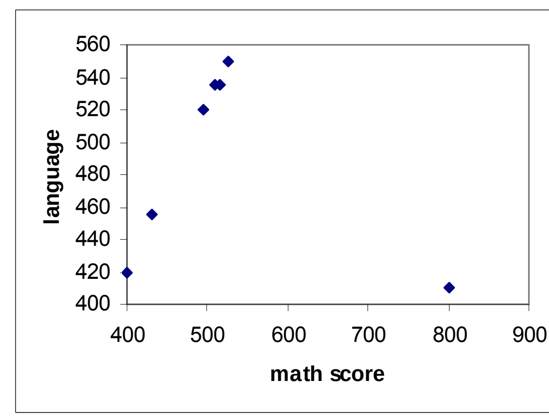
r=-0.21



r=0.74



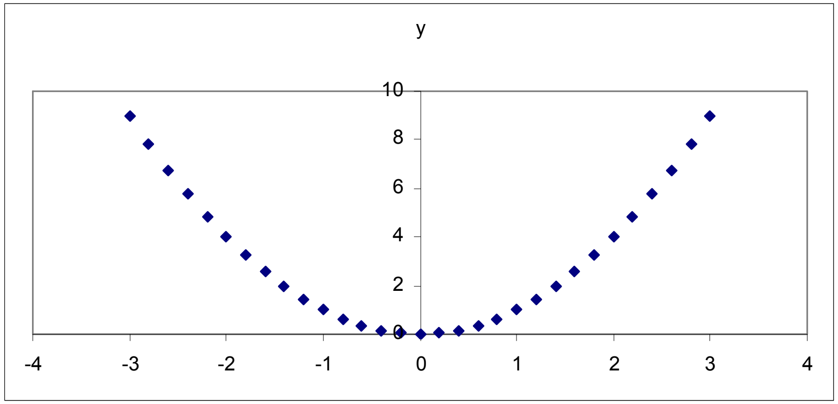
r=0.998



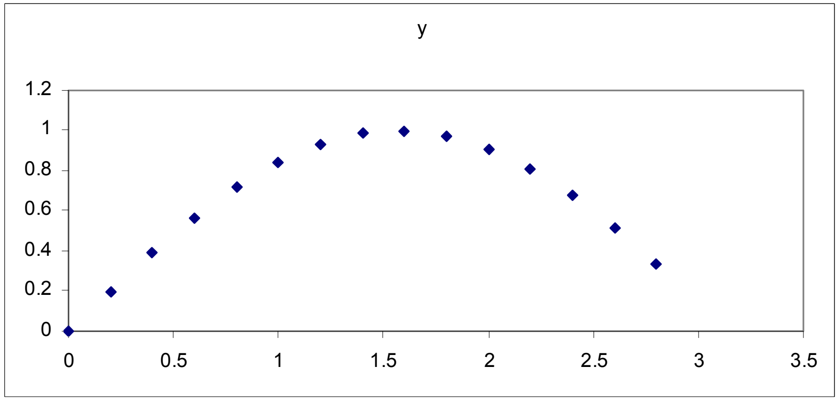
r=-0.26

Correlation and linearity

- Two variables may be closely related and still have a small correlation if the form of the relationship is not linear.

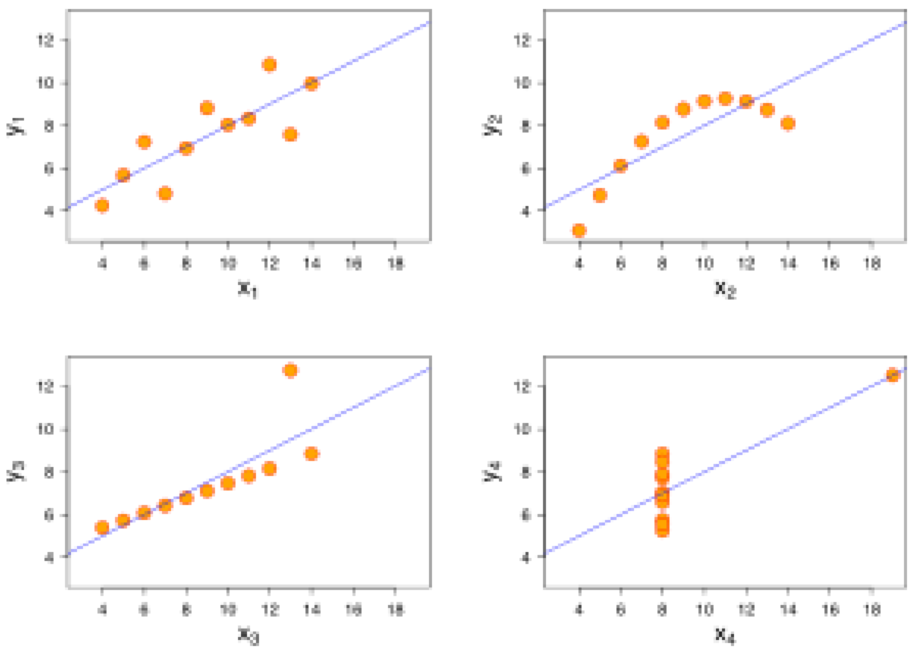


***r*=2.8 E-15 (=0.0000000000000028)**



***r*=0.157**

Correlation and linearity



Four sets of data with the same correlation of 0.816

http://en.wikipedia.org/wiki/Correlation_and_dependence

Coefficient of determination

- The square of the correlation coefficient multiplied by 100 is called the coefficient of determination.
- It shows the percentages of the total variation explained by the linear regression.
- **Example.**
- The correlation between math aptitude and language aptitude was found r =0,9989.
  The coefficient of determination, r^2 = 0.917 .
  So 91.7% of the total variation of Y is caused by its linear relationship with X .

When is a correlation „high”?

- What is considered to be high correlation varies with the field of application.
- The statistician must decide when a sample value of *r* is far enough from zero, that is, when it is sufficiently far from zero to reflect the correlation in the population.

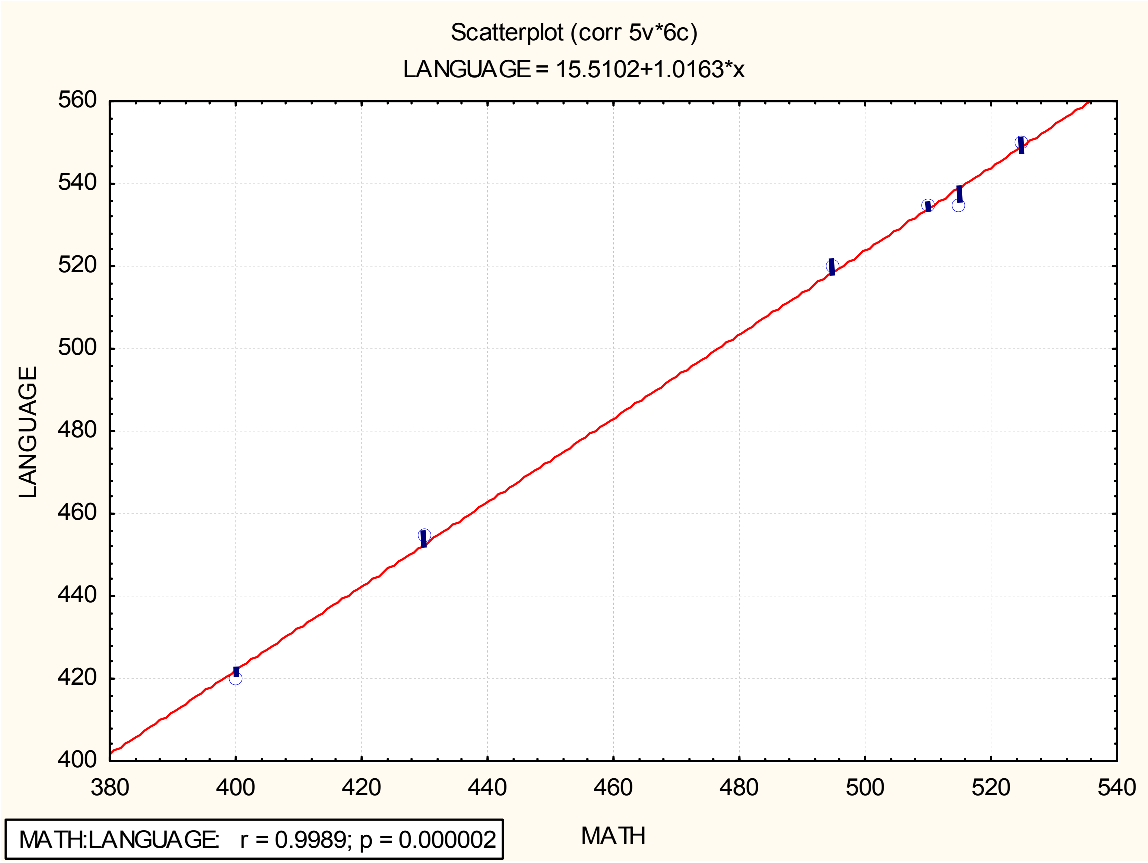
Testing the significance of the coefficient of correlation

- The statistician must decide when a sample value of r is far enough from zero to be significant, that is, when it is sufficiently far from zero to reflect the correlation in the population.
- (details: lecture 8.)

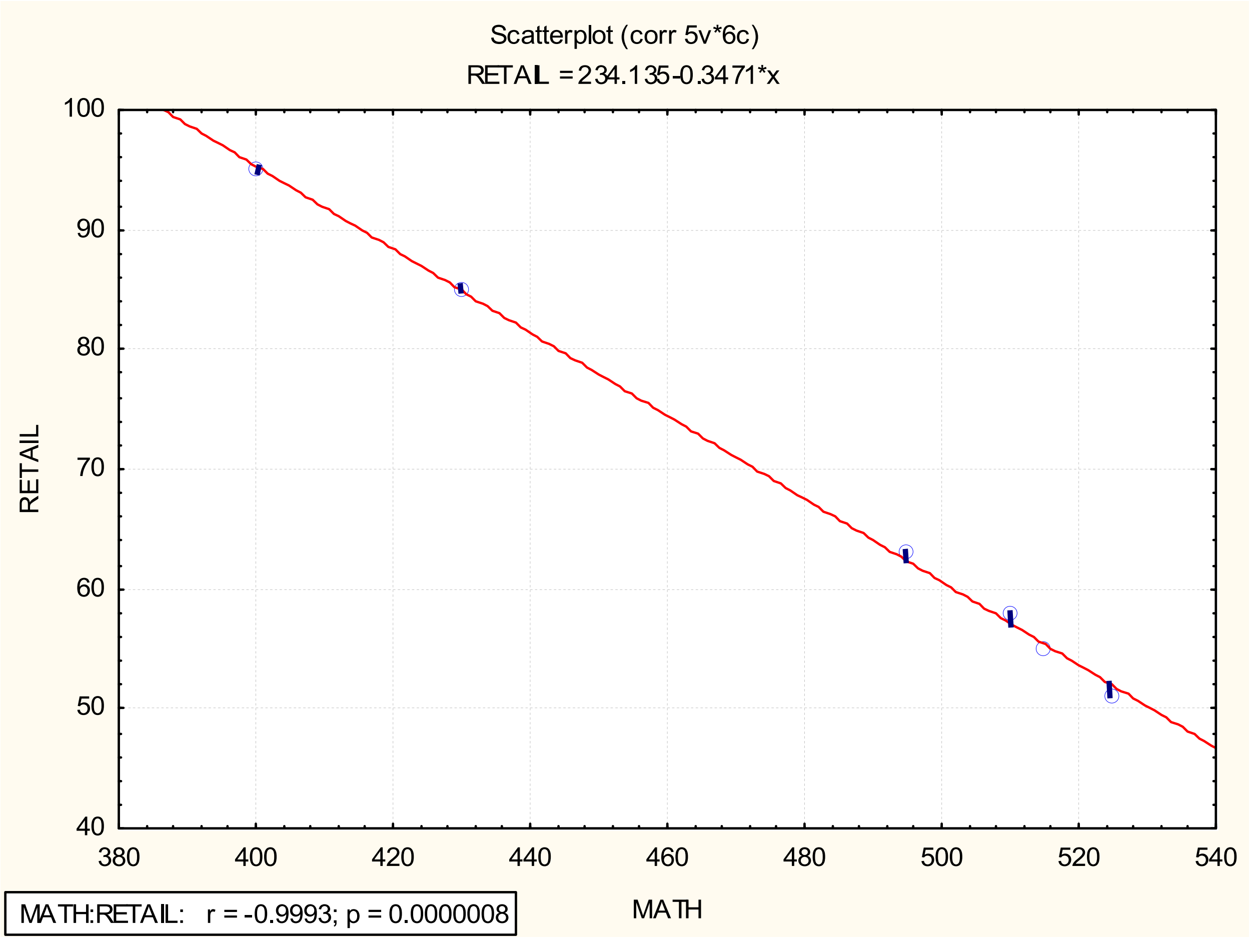
Prediction based on linear correlation: the linear regression

- When the form of the relationship in a scatterplot is linear, we usually want to describe that linear form more precisely with numbers.
- We can rarely hope to find data values lined up perfectly, so we fit lines to scatterplots with a method that compromises among the data values. This method is called the method of least squares.
- The key to finding, understanding, and using least squares lines is an understanding of their failures to fit the data; the residuals.

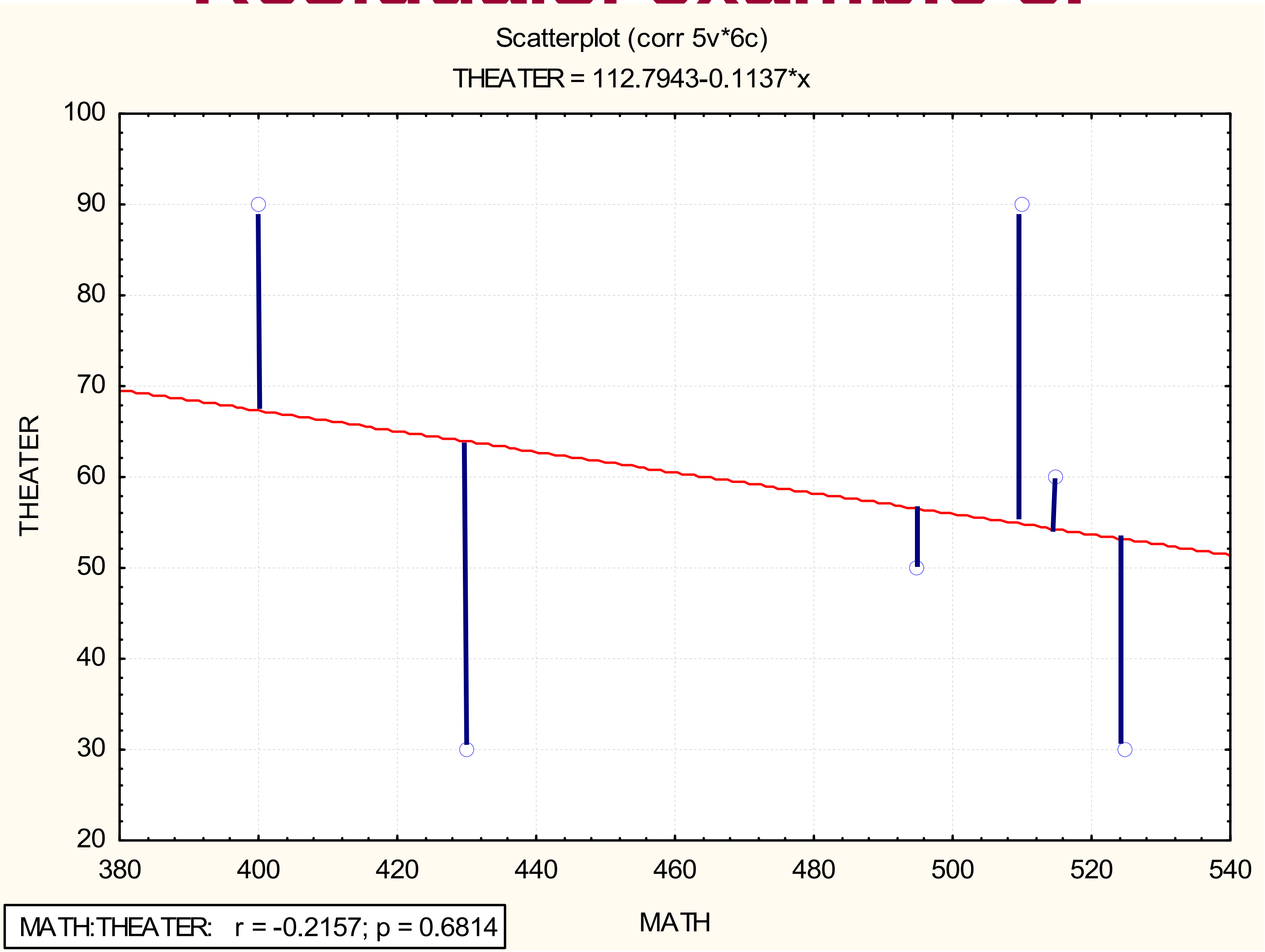
Residuals, example 1.



Residuals, example 2.



Residuals. example 3.



Prediction based on linear correlation: the linear regression

- A straight line that best fits the data:

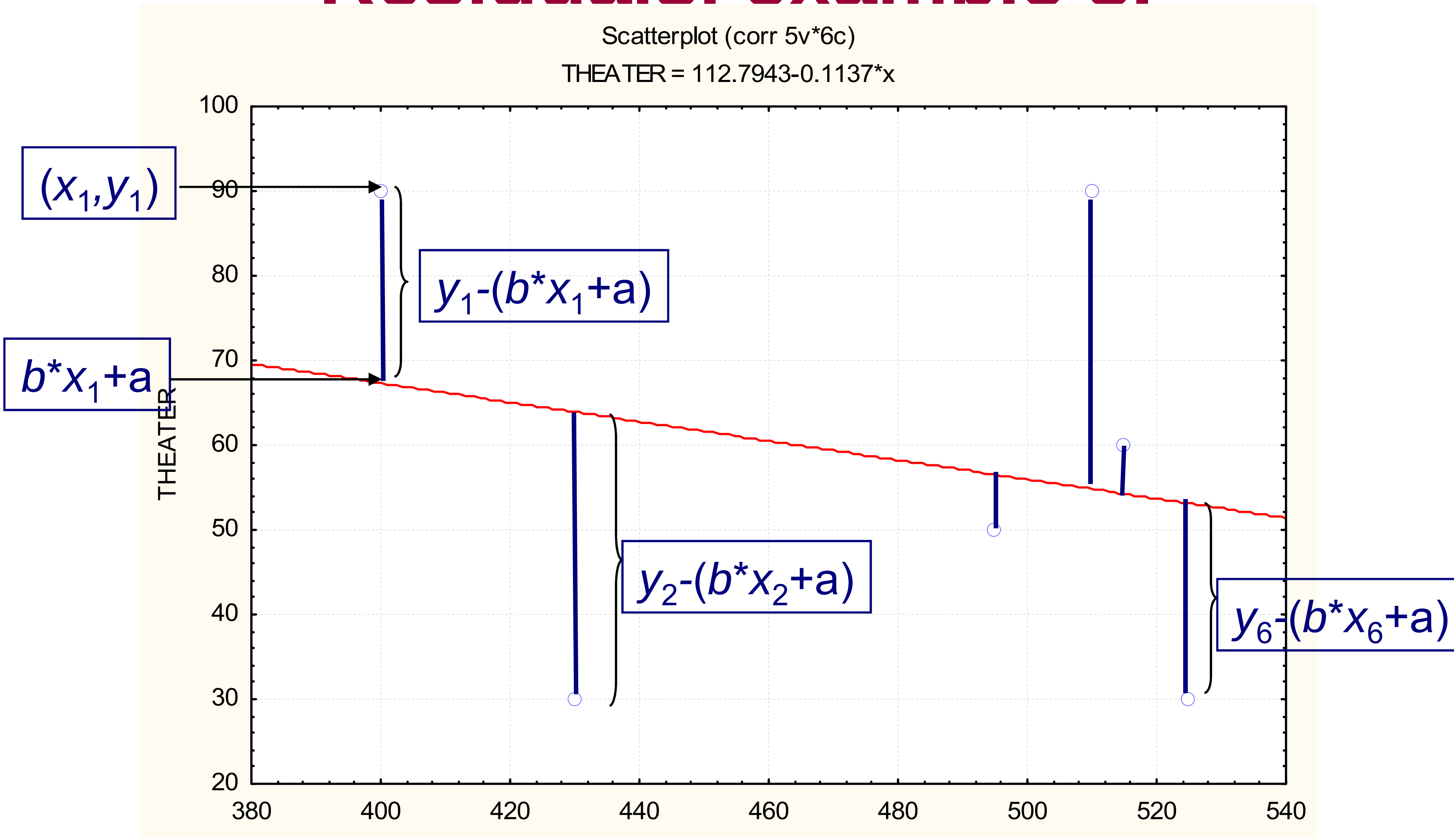
$$y=bx + a \quad or \quad y= a + bx$$

  is called regression line
- **Geometrical meaning of *a* and *b*.**
- *b*: is called regression coefficient, slope of the best-fitting line or regression line;
- *a*: *y*-intercept of the regression line.

- The principle of finding the values ***a*** and ***b*,** given $x_1,x_2,\ldots x_n$ and $y_1,y_2,\ldots y_n$.
- Minimising the sum of squared residuals, i.e.

$$\Sigma(y_i-(a+bx_i))^2 \rightarrow \min$$

Residuals. example 3.



The general equation of a line is y = a + b x. We would like to find the values of *a* and *b* in such a way that the resulting line be the best fitting line. Let's suppose we have *n* pairs of (x_i, y_i) measurements. We would like to approximate y_i by values of a line . If x_i is the independent variable, the value of the line is $a + b\,x_i$.

We will approximate y_i by the value of the line at x_i, that is, by $a + b\,x_i$. The approximation is good if the differences $y_i - (a + b \cdot x_i)$ are small. These differences can be positive or negative, so let's take its square and summarize:

$$\sum_{i=1}^{n} (y_i - (a + b \cdot x_i))^2 = S(a,b)$$

This is a function of the unknown parameters *a* and *b*, called also the sum of squared residuals. To determine *a* and *b*: we have to find the minimum of S(*a,b*). In order to find the minimum, we have to find the derivatives of S, and solve the equations

$$\frac{\partial S}{\partial a} = 0, \quad \frac{\partial S}{\partial b} = 0$$

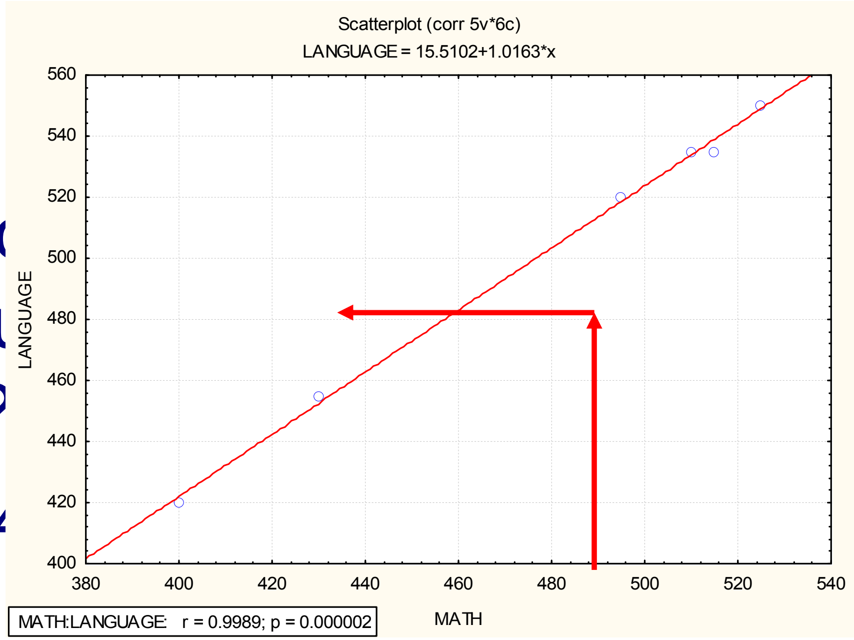
The solution of the equation-system gives the formulas for *b* and *a*:

$$b = \frac{n \cdot \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n \cdot \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \text{ and } a = \bar{y} - b \cdot \bar{x}$$

It can be shown, using the 2nd derivatives, that these are really minimum places.

Equation of regression line for the data of Example 1.

- $y = 1.016 \cdot x + 15.5$
  the slope of the line is 1.01
- Prediction based on the equation: what is the predi score for language for a stu having 400 points in math?
- $y_{predicted} = 1.016 \cdot 400 + 15.5 = 4$



Computation of the correlation coefficient from the regression coefficient.

- There is a relationship between the correlation and the regression coefficient:

$$r = b \cdot \frac{s_x}{s_y}$$

- where s_x, s_y are the standard deviations of the samples .
- From this relationship it can be seen that the sign of *r* and *b* is the same: if there exist a negative correlation between variables, the slope of the regression line is also negative .

SPSS output for the relationship between age and body mass

Model Summary

R	R Square	Adjusted R Square	Std. Error of the Estimate
.018	.000	-.007	13.297

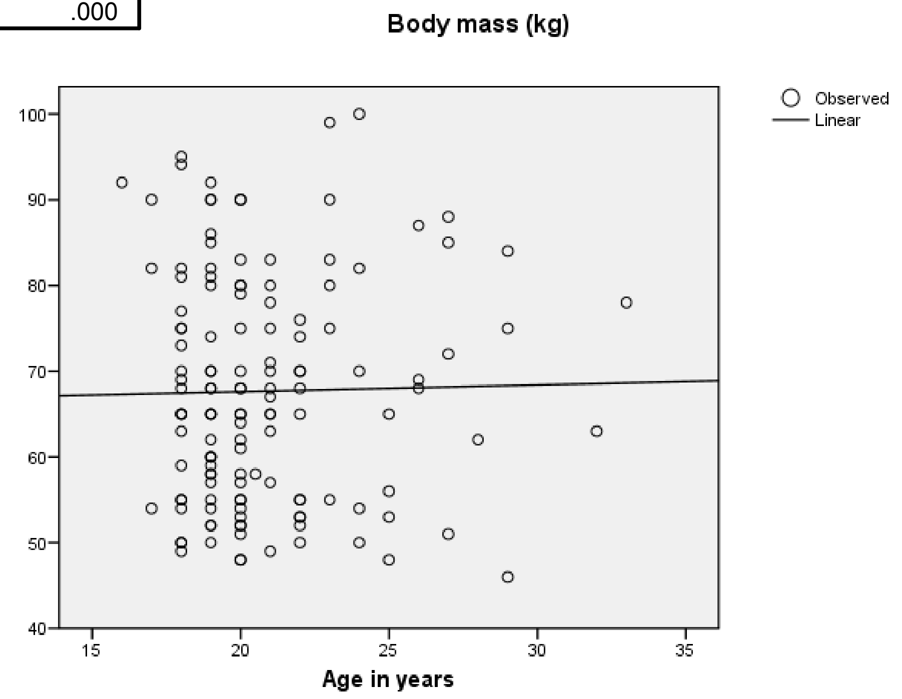
The independent variable is Age Age in years.

Coefficient of correlation, r=0.018

Coefficients

	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
Age Age in years	.078	.372	.018	.211	.833
(Constant)	66.040	7.834		8.430	.000

Equation of the regression line:
$y=0.078x+66.040$



SPSS output for the relationship between body mass at present and 3 years ago

Model Summary

R	R Square	Adjusted R Square	Std. Error of the Estimate
.873	.763	.761	5.873

The independent variable is Mass Body mass (kg).

Coefficient of correlation, r=0.873

Coefficients

	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
Mass Body mass (kg)	.795	.039	.873	20.457	.000
(Constant)	10.054	2.670		3.766	.000

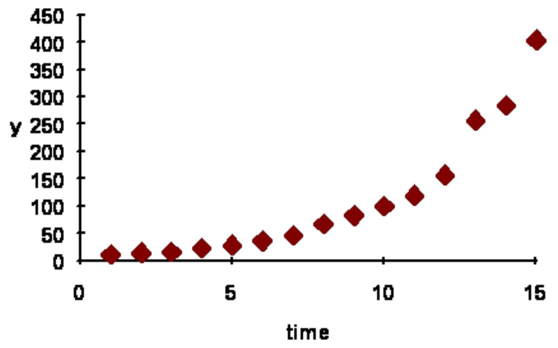
Equation of the regression line:
y=0.795x+10.054

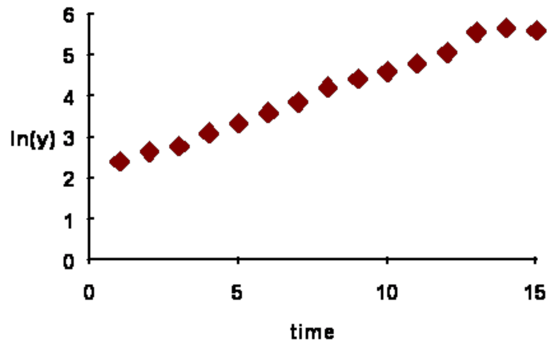
Regression using transformations

- Sometimes, useful models are not linear in parameters. Examining the scatterplot of the data shows a functional, but not linear relationship between data.

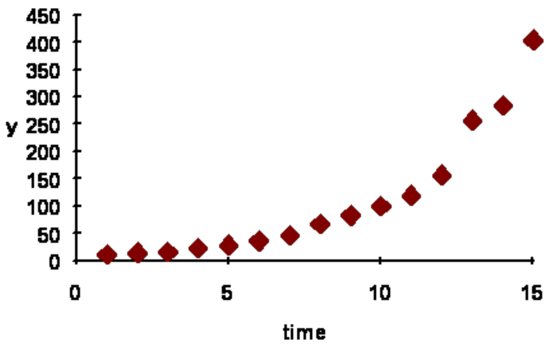
Example

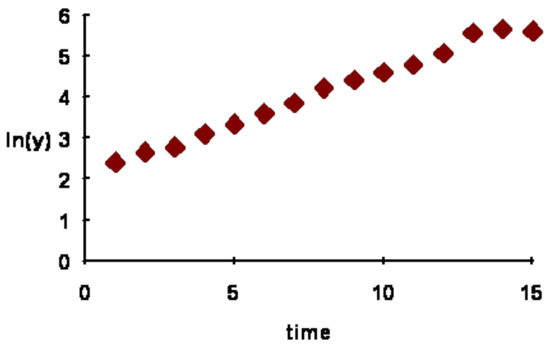
- A fast food chain opened in 1974. Each year from 1974 to 1988 the number of steakhouses in operation is recorded.
- The scatterplot of the original data suggests an exponential relationship between x (year) and y (number of Steakhouses) (first plot)
- Taking the logarithm of y, we get linear relationship (plot at the bottom)





- Performing the linear regression procedure to x and log (y) we get the equation
- log $y = 2.327 + 0.2569\,x$
- that is
- $y = e^{2.327 + 0.2569\,x} = e^{2.327}e^{0.2569x} = 1.293e^{0.2569x}$ is the equation of the best fitting curve to the original data.





$y = 1.293e^{0.2569x}$

$\log y = 2.327 + 0.2569\, x$

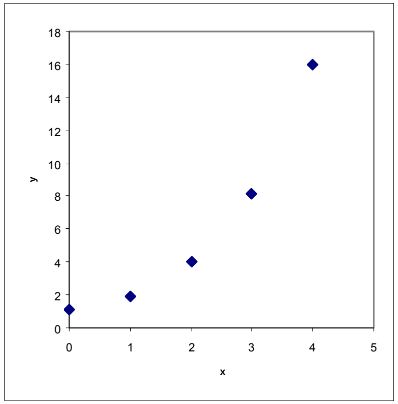
Types of transformations

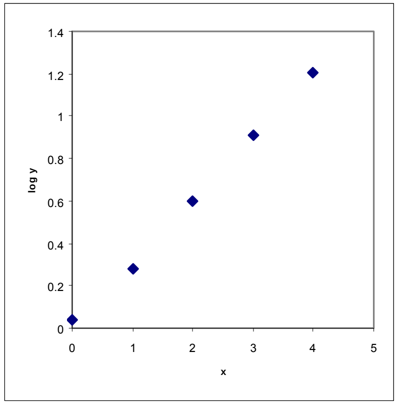
- Some non-linear models can be transformed into a linear model by taking the logarithms on either or both sides. Either 10 base logarithm (denoted log) or natural (base e) logarithm (denoted ln) can be used. If $a>0$ and $b>0$, applying a logarithmic transformation to the model

Exponential relationship ->take log y

x	y	lg y
0	1.1	0.041393
1	1.9	0.278754
2	4	0.60206
3	8.1	0.908485
4	16	1.20412

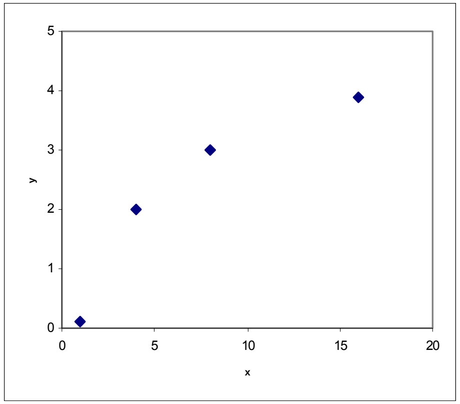
- Model: $y=a*10^{bx}$
- Take the logarithm of both sides:
- lg y =lg$a$+bx
- so lg y is linear in x

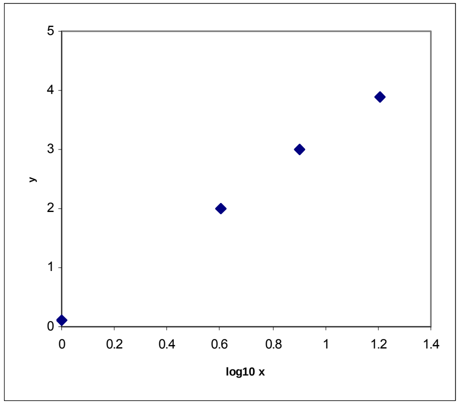




Logarithm relationship ->take log x

x	y	log x
1	0.1	0
4	2	0.60206
8	3.01	0.90309
16	3.9	1.20412

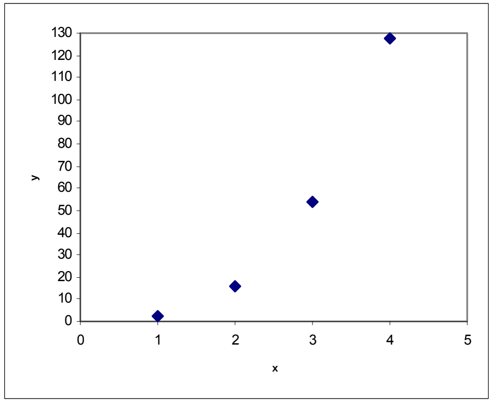




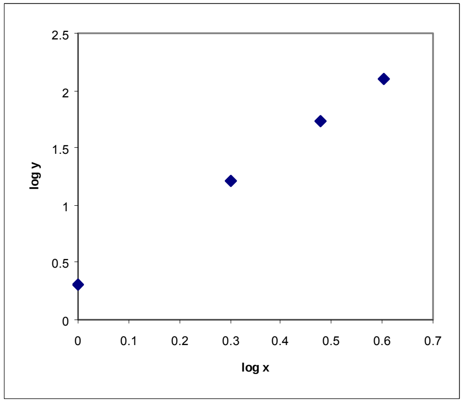
- Model: $y=a+\lg x$
- so y is linear in $\lg x$

Power relationship ->take log x and log y

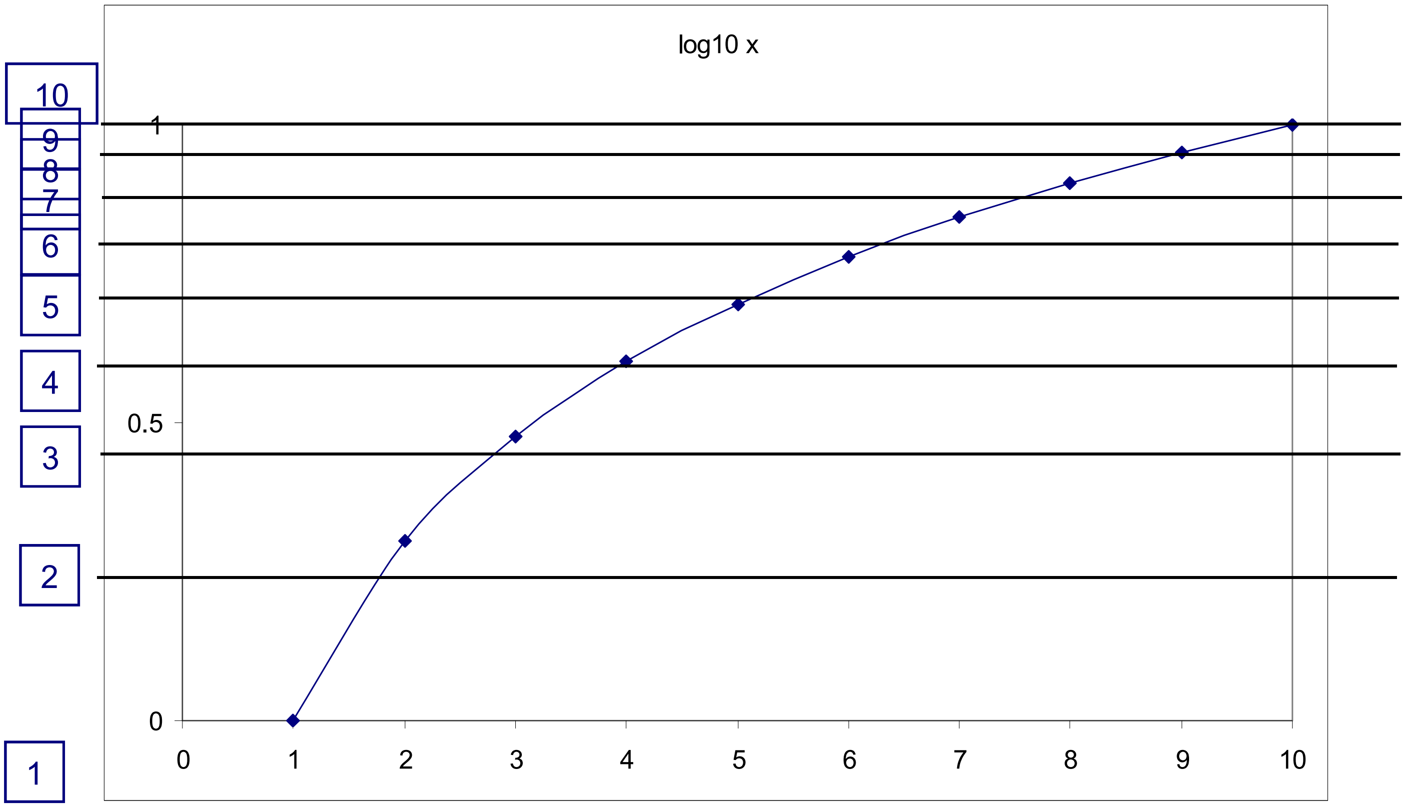
x	y	log x	log y
1	2	0	0.30103
2	16	0.30103	1.20412
3	54	0.477121	1.732394
4	128	0.60206	2.10721



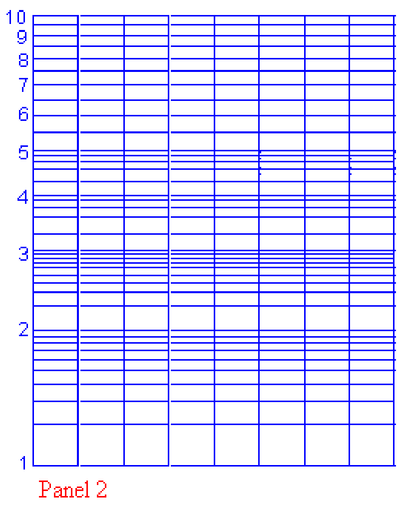
- Model: $y=ax^b$
- Take the logarithm of both sides:
- lg y =lg$a$+b lg$x$
- so lg$y$ is linear in lg x

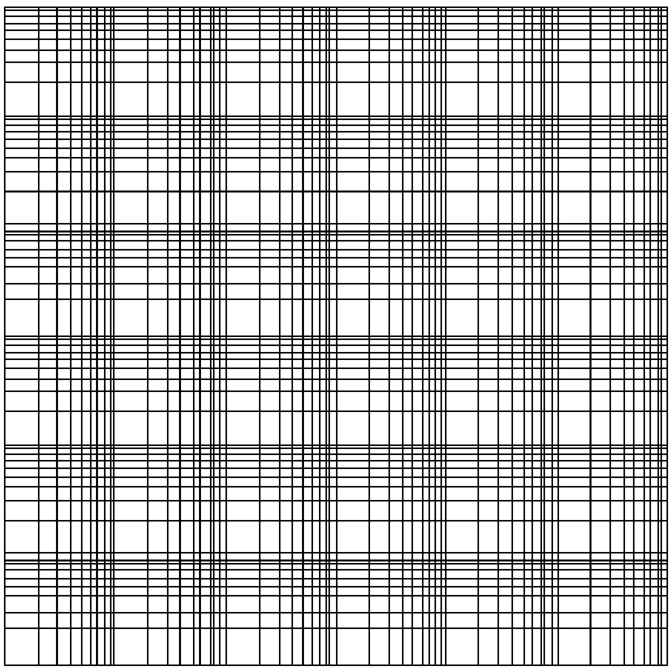


Log10 base logarithmic scale



Logarithmic papers





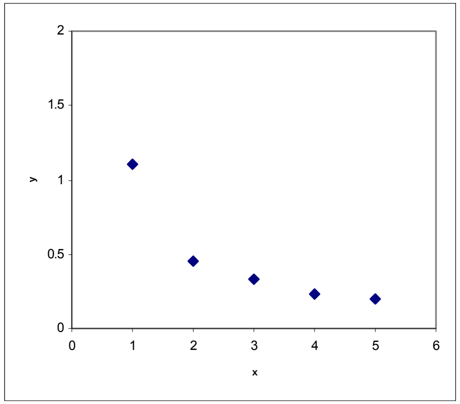
Semilogarithmic paper

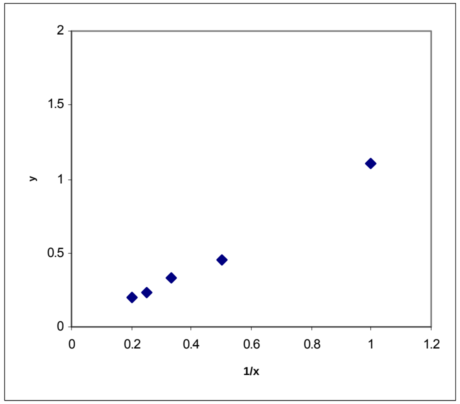
log-log paper

Reciprocal relationship ->take reciprocal of x

x	y	1/x
1	1.1	1
2	0.45	0.5
3	0.333	0.333333
4	0.23	0.25
5	0.1999	0.2

- Model: $y=a+b/x$
- $y=a+b*1/x$
- so y is linear in $1/x$





Example from the literature

Circulation

JOURNAL OF THE AMERICAN HEART ASSOCIATION

American Heart Association

Learn and Live

Correlation between echocardiographic endocardial surface mapping of abnormal wall motion and pathologic infarct size in autopsied hearts

GT Wilkins, JF Southern, CY Choong, JD Thomas, JT Fallon, DE Guyer and AE Weyman

Circulation 1988;77;978-987

Circulation is published by the American Heart Association. 7272 Greenville Avenue, Dallas, TX 72514

 Print ISSN: 0009-7322. Online ISSN: 1524-4539

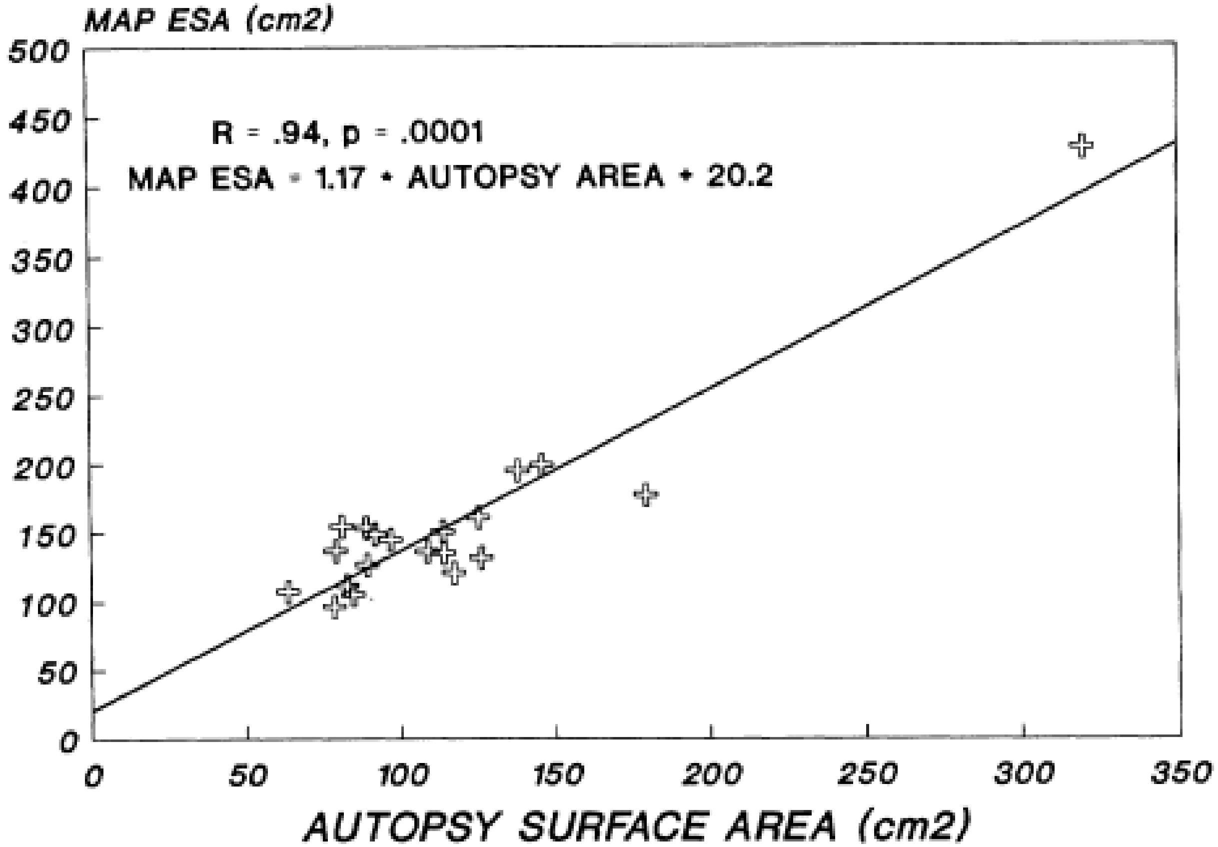
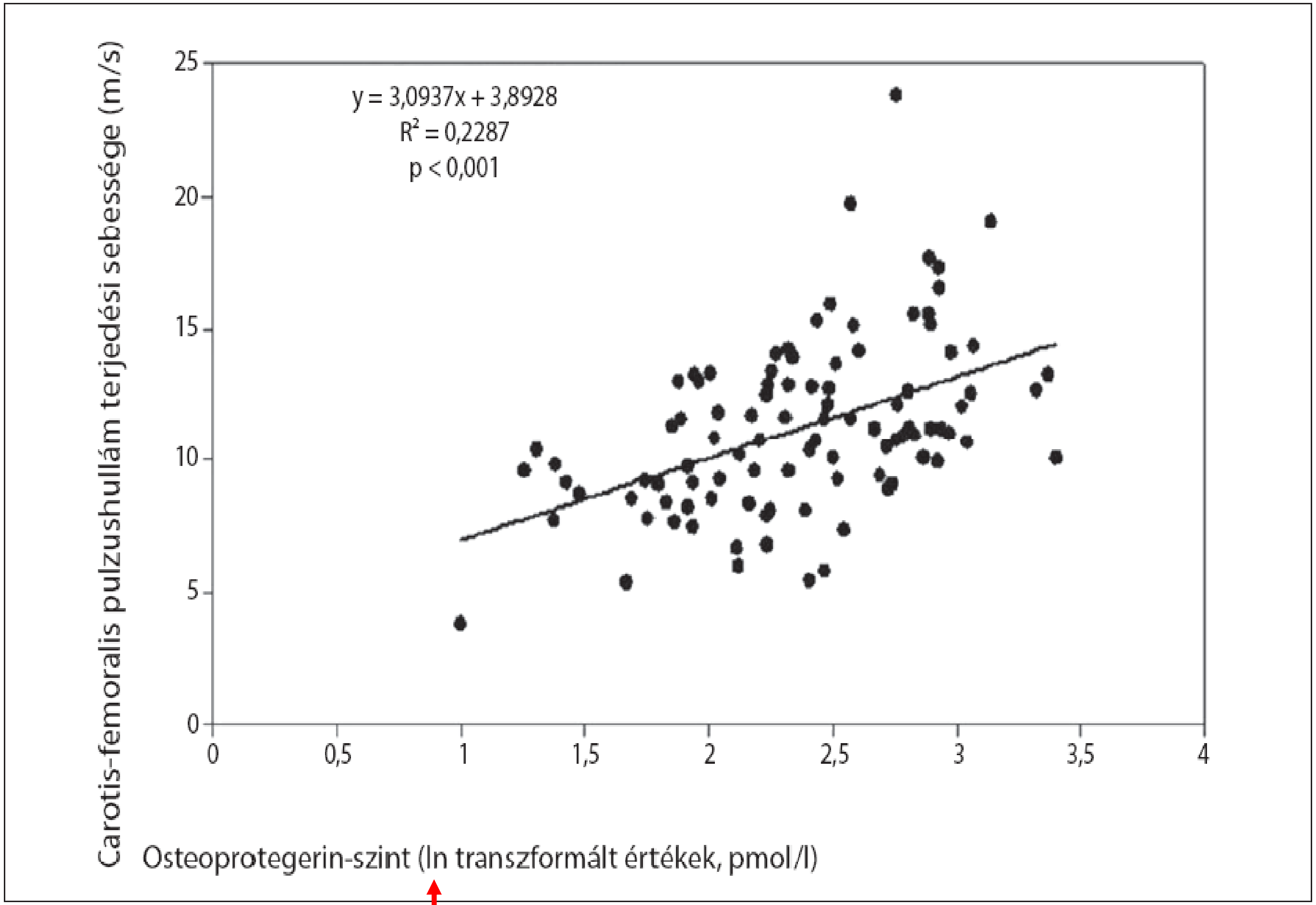


FIGURE 4. Correlation of the left ventricular endocardial surface area measured at autopsy (Autopsy Surface Area) with the endocardial surface area derived from the echocardiographic map (MAP ESA).

Example 2. EL HADJ OTHMANE TAHA és mtsai: Osteoprotegerin: a regulátor, a protektor és a marker. Orvosi Hetilap 2008 ■ 149. évfolyam, 42. szám ■ 1971–1980.



3. ábra | A carotis-femoralis PWV és az osteoprotegerin szérumszintje közötti lineáris regresszió

Useful WEB pages

- http://davidmlane.com/hyperstat/desc_biv.html
- http://onlinestatbook.com/stat_sim/reg_by_eye/index.html
- http://www.youtube.com/watch?v=CSYTZWFnVpg&feature=related
- http://www.statsoft.com/textbook/basic-statistics/#Correlationsb
- http://people.revoledu.com/kardi/tutorial/Regression/NonLinear/LogarithmicCurve.htm
- http://www.physics.uoguelph.ca/tutorials/GLP/

The origin of the word „regression”. Galton: Regression towards mediocrity in hereditary stature. Journal of the Anthropological Institute 1886 Vol.15, 246-63

TABLE I.

NUMBER OF ADULT CHILDREN OF VARIOUS STATURES BORN OF 205 MID-PARENTS OF VARIOUS STATURES.
(All Female heights have been multiplied by 1·08).

Heights of the Mid-parents in inches.	Heights of the Adult Children.														Total Number of		Medians.
	Below	62·2	63·2	64·2	65·2	66·2	67·2	68·2	69·2	70·2	71·2	72·2	73·2	Above	Adult Children.	Mid-parents.	
Above ..	..	..	..	..	..	..	..	..	..	..	..	1	3	..	4	5	..
72·5	..	..	..	..	..	..	..	1	2	1	2	7	2	4	19	6	72·2
71·5	..	..	..	..	1	3	4	3	5	10	4	9	2	2	43	11	69·9
70·5	1	..	1	..	1	1	3	12	18	14	7	4	3	3	68	22	69·5
69·5	..	..	1	16	4	17	27	20	33	25	20	11	4	5	183	41	68·9
68·5	1	..	7	11	16	25	31	34	48	21	18	4	3	..	219	49	68·2
67·5	..	3	5	14	15	36	38	28	38	19	11	4	..	..	211	33	67·6
66·5	..	3	3	5	2	17	17	14	13	4	..	..	..	..	78	20	67·2
65·5	1	..	9	5	7	11	11	7	7	5	2	1	..	..	66	12	66·7
64·5	1	1	4	4	1	5	5	..	2	..	..	..	..	..	23	5	65·8
Below ..	1	..	2	4	1	2	2	1	1	..	..	..	..	..	14	1	..
Totals ..	5	7	32	59	48	117	138	120	167	99	64	41	17	14	928	205	..
Medians ..	..	..	66·3	67·8	67·9	67·7	67·9	68·3	68·5	69·0	69·0	70·0	..	..	..	..	..

NOTE.—In calculating the Medians, the entries have been taken as referring to the middle of the squares in which they stand. The reason why the headings run 62·2, 63·2, &c., instead of 62·5, 63·5, &c., is that the observations are unequally distributed between 62 and 63, 63 and 64, &c., there being a strong bias in favour of integral inches. After careful consideration, I concluded that the headings, as adopted, best satisfied the conditions. This inequality was not apparent in the case of the Mid-parents.

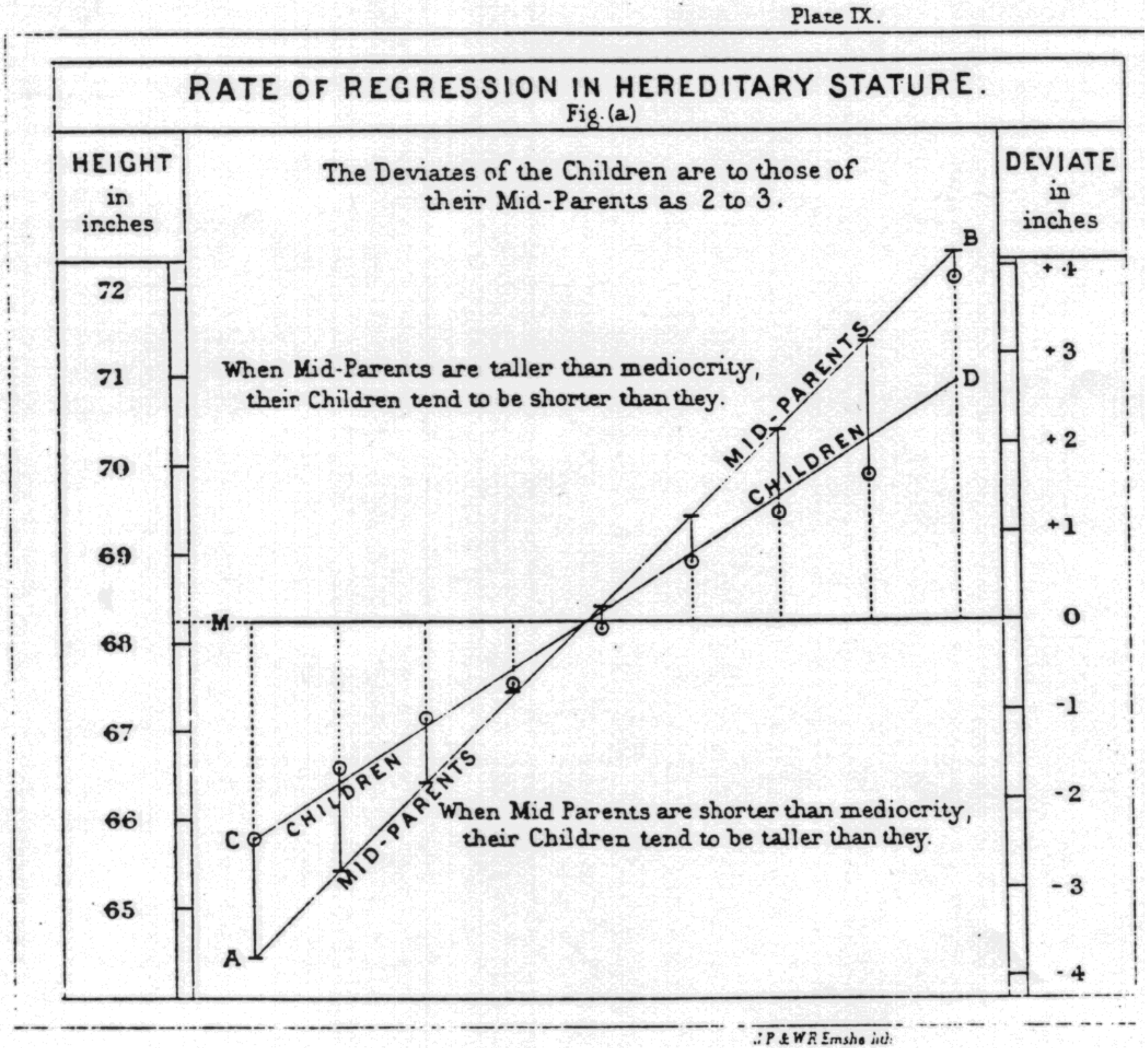


Plate IX.
RATE OF REGRESSION IN HEREDITARY STATURE.
Fig. (a)
HEIGHT in inches
DEVIATE in inches
The Deviates of the Children are to those of their Mid-Parents as 2 to 3.
When Mid-Parents are taller than mediocrity, their Children tend to be shorter than they.
When Mid Parents are shorter than mediocrity, their Children tend to be taller than they.
MID-PARENTS
CHILDREN
72
71
70
69
68
67
66
65
+4
+3
+2
+1
0
-1
-2
-3
-4
A
B
C
D
M