2. Elementary functions, inverse function

PROBLEM 1 INVERSE FUNCTION, HORIZONTAL LINE TEST

- Is the function invertible? If not, find a subinterval of the domain on which it is partially invertible. Sketch the graph of the function and its inverse.

a) $y = x^2$ b) $y = x^3$ c) $y = x^{-2}$ d) $y = \sqrt[-2]{x}$ e) $y = \sqrt[-3]{x}$

PROBLEM 2 FINDING THE INVERSE FUNCTIONS' FORMULA

- Determine the formula for the inverse of the following functions:

a) $y = f(x) = \dfrac{2x-1}{3x+2}$ b) $y = f(x) = 2x^3 + 1$ c) $y = f(x) = \sqrt[3]{x+4}$

d) $y = f(x) = \dfrac{x^3}{x^3+1}$ e) $y = f(x) = \dfrac{1}{\sqrt[5]{x^3+1}}$ f) $y = f(x) = \dfrac{\sqrt[3]{x}-1}{\sqrt[3]{x}}$

g) $y = f(x) = 1 + 2\ln(x+1)$ h) $y = f(x) = \dfrac{e^x}{e^x+1}$ i) $y = f(x) = \log_3(3+3^x)$

PROBLEM 3 FINDING THE PARTIAL INVERSE

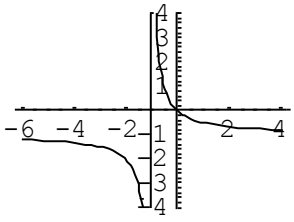
- On which subinterval is the function invertible? Find the formula of the partial inverse and make a graph:

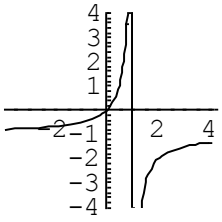
a) $y = f(x) = 1 - x^2$ b) $y = f(x) = 2x^2 - 5x$ c) $y = f(x) = x^2 - 2x + 3$

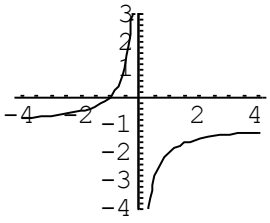
PROBLEM 4 GRAPHICAL PROBLEMS

- Choose the inverse of $f(x)$ of the graphs below:

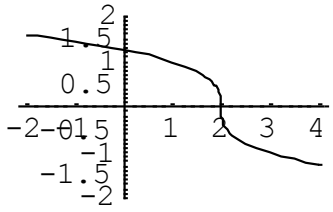
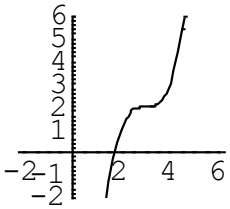
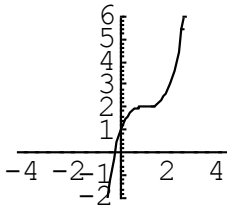
$$f(x\_) := \frac{x}{x+1};$$



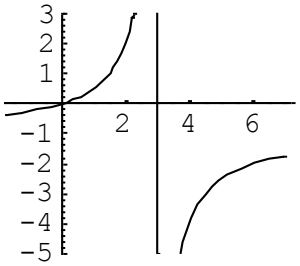
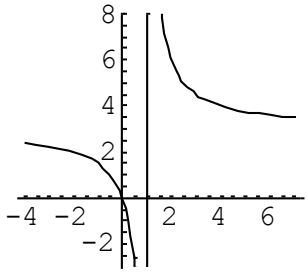
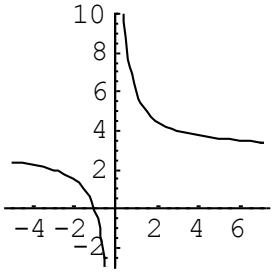




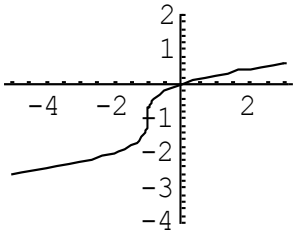
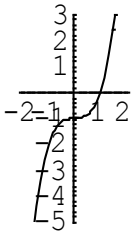
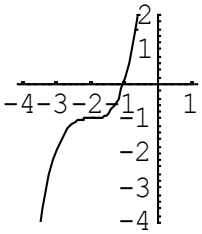
$$f(x\_) := \sqrt[3]{x-2}+1;$$

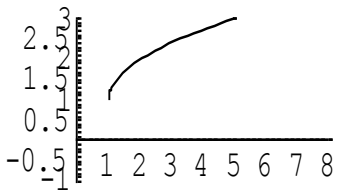
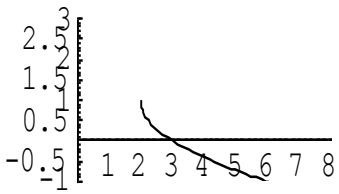
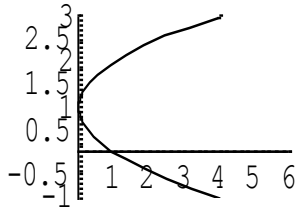
$$f(x\_) := \frac{x}{x-3};$$

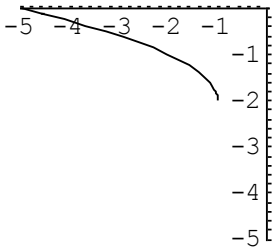
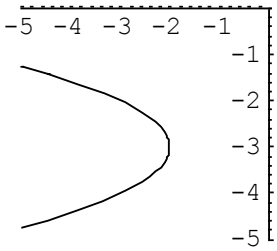
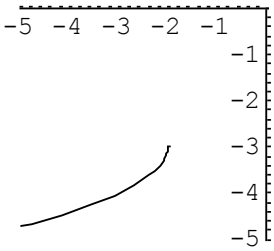
$$f(x\_) := \sqrt[3]{x+1}-2;$$

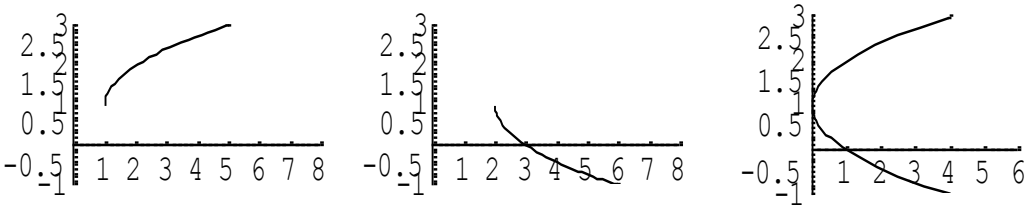
$$f(x\_) := (x-1)^2+2;$$

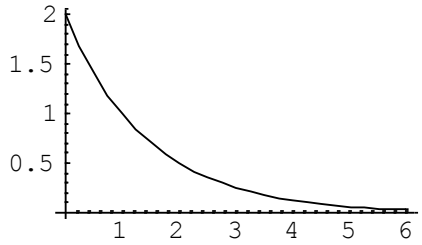
$$f(x\_) := -(x+3)^2-2;$$

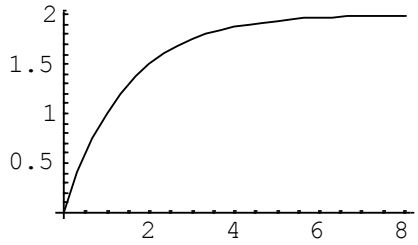
$f(x\_) := (x-1)^2 + 2;$



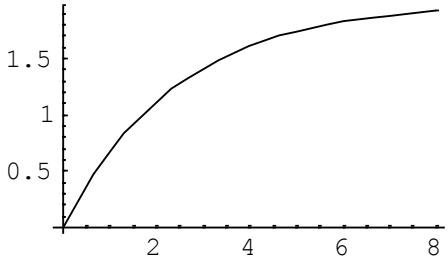
- Find the value of the inverse of $f(x)$ at the places y_1 and y_2:



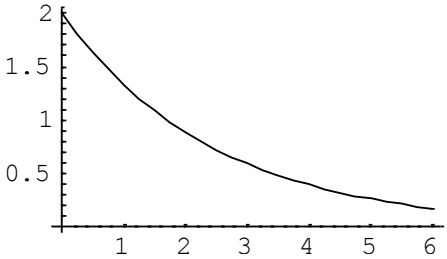
$y_1 = 1;\ y_2 = 1.5;$



$y_1 = 0.5;\ y_2 = 1.5;$



$y_1 = 0.5;\ y_2 = 1;$



$y_1 = 0.5;\ y_2 = 1;$

PROBLEM 5 COMPOSITE FUNCTION

- Give the formula for the composite function $f(g(x))$:

  a) $f(x) = x \sin x$, $g(x) = 2x^3 - 1$

  b) $f(x) = 2x^2 - x + 3$, $g(x) = 2^x$

  c) $f(x) = x + \lg x$, $g(x) = x^{-1}$

- Give nontrivial formulas for the outer function $f(x)$ and inner function $g(x)$:

  a) $f(g(x)) = (1+\cos x)^2$ b) $f(g(x)) = x^3 + \lg x^3$ c) $f(g(x)) = x^4 \tan x^2$

PROBLEM 6 GRAPHING ELEMENTARY FUNCTIONS

- Sketch the graph of the below **power function** and give its domain (of definition) and range (of values)! Whenever defined, mark the values at $x = -1, 0, 1$.

  a) $y = x^2$ b) $y = x^3$ c) $y = x^{-1}$ d) $y = x^{-2}$ e) $y = x^{-3}$

  f) $y = x^{1/2}$ g) $y = x^{1/3}$ h) $y = x^{2/3}$ i) $y = x^{-1/2}$ j) $y = x^{-2/3}$

  k) $y = x^{3/2}$ l) $y = x^{\pi}$

- Sketch the graph of the below **exponential function** and its inverse (**logarithmic function**) and specify their domain and range! Whenever defined, mark the values at $x = -1, 0, 1$.

  a) $y = 2^x$ b) $y = \left(\frac{1}{2}\right)^x$ c) $y = 3^x$ d) $y = \left(\frac{1}{3}\right)^x$ e) $y = e^x$

  , e=2.71828

- Sketch the graph of the below **trigonometric function** and give its domain and range! Whereever defined, mark the values at $x = -\pi/2, -\pi/4, 0, \pi/4, \pi/2, \pi, 3\pi/2, 2\pi$.

  a) $y = \sin x$ b) $y = \cos x$ c) $y = \text{tg}\, x$ d) $y = \text{ctg}\, x$