1. Elementary problems

1.1. Concentration of a solution

If a solution of mass m contains dissolved material (solute) of mass μ, then the *concentration* of the solution is: $c = \frac{\mu}{m}$. When specifying a solution generally the total mass (m) and the concentration expressed in percents (c=100c%) are given, from these, however, it is easy to calculate the mass of *solute* (dissolved material): $\mu = c \cdot m$, as well as the mass of *solvent*: $m - c \cdot m = (1-c)m$. **Ex.:** in 2 kg salt solution of concentration 15% (m=2 kg, c=15%=15/100=0.15) the mass of dissolved salt is: $cm = 0.15 \cdot 2$ kg$=0.3$ kg, the mass of water (solvent) is: 2 kg$-$0.3 kg$=$1.7 kg.

Concentration of a mixture. When mixing two solutions we can use the following notation:

	total mass	**concentration**	**mass of solute**
1st solution:	m_1	c_1	$c_1\, m_1$
2nd solution:	m_2	c_2	$c_2\, m_2$
mixture:	m	c	$c\, m$

By the law of mass conservation the masses add up when mixing, therefore $m = m_1 + m_2$, and $cm = c(m_1+m_2) = c_1 m_1 + c_2 m_2$, from which we can calculate the *concentration of the mixture*:

$$c = \frac{c_1 m_1 + c_2 m_2}{m_1 + m_2}$$

PROBLEM 1 CONCENTRATION OF SOLUTIONS, MIXTURE PROBLEMS

- How much solute (dissolved material) and solvent is needed to make 8 kg 30% solution?
- What is the concentration of 15 kg solution containing 8 kg solute?
- How much water should be added to 6 kg salt, to prepare a 25% concentrated solution?
- How much salt should be dissolved in 700 g water to obtain a 15% solution?
- We have 1.5 kg 90% concentrated alcohol. How much water do we need to dilute it to 50%?
- After adding salt to 4.5 kg 20% solution, the concentration rises to 30%. What will be the mass of the obtained 30% solution?
- We mix 3 kg 40% solution with 5 kg 12% solution. What will be the concentration of the mixture?

- We have 2.5 kg 5% solution. How much 20% solution should be mixed to it to get a 12% concentrated solution?
- What amounts of 10% and 25% solutions should be mixed to get 2 kg 20% solution?
- What amounts of 10% and 25% solutions should be mixed to get 3.5 kg 35% solution?
- After mixing 250 g 25% solution with 100 g solution of unknown concentration we obtain a 20% concentrated mixture. What was the unknown concentration?
- In what ratio should we mix 15% and 40% concentrated solutions in order to obtain a 30% concentrated mixture?

RESULTS

1. 2.4 kg solute has to be dissolved in 5.6 kg solvent **2.** c=8/15=53.3% **3.** 18 kg water **4.** 123.5 g salt **5.** 1.2 kg water **6.** m=5.14 kg **7.** c=0.225=22.5% **8.** We have to add 0.9375 kg of 20% concentrated solution **9.** we have to mix 2/3 kg of 10% and 4/3 kg of 25% concentration **10.** no solution (impossible) **11.** the unknown concentration was: 0.075=7.5% **12.** we mix in a ratio of 2÷3, that is, 2 parts of the 15% concentrated with 3 parts of the 40% concentrated solution

1.2. Equations of straight lines – linear (1st degree) functions

Normal equation of a straight line: $y = a\,x + b$, where a = slope, b = y–intercept

Factorized equation of a straight line: $y = a\,(x - x_0)$, where $x_0 = -\frac{b}{a}$ = x–intercept, root (zero)

Equation of the straight line passing through the point (x_1, y_1) **with slope** a**:**

$y = a\,(x - x_1) + y_1$

Equation of a straight line passing through two given points $(x_1,\ y_1)$ **and** $(x_2,\ y_2)$**:**

first calculate the slope: $a = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$,

then substitute into the previous point–slope equation.

Given two straight lines' normal equations: $y = a_1\,x + b_1$ and $y = a_2\,x + b_2$ (a_1, b_1, a_2, b_2 given);

the two lines are *parallel*, if $a_1 = a_2$, are *perpendicular*, if $a_1\,a_2 = -1$, furthermore the coordinates of their *intersection point* are the two solutions of the system of their equations with two unknowns (x and y).

PROBLEM 2 EQUATIONS OF STRAIGHT LINES – LINEAR (1^{st} DEGREE) FUNCTIONS

- Give the equation of the straight line passing through the given point (x_1,y_1) with slope a, determine the x- and y-intercepts, and draw the graph of the line including all points involved (the given point and the two intercept points):

  a) $(x_1, y_1)=(0,0)$ and $a = 2$ *b*) $(x_1, y_1)=(0,1)$ and $a = -2$

  c) $(x_1, y_1)=(-3,2)$ and $a = -½$ *d*) $(x_1, y_1)=(10,0)$ and $a = -3$

  e) $(x_1, y_1)=(1,1)$ and $a = -1$ *f*) $(x_1, y_1)=(-3,-2)$ and $a = ¾$

- Determine the equation and the intercepts of the straight line passing through the two given points and make a graph:

  a) (0,0) and (–3,–2) *b*) (0,10) and (10,0) *c*) (–1,-1) and (2,3)

  d) (3,4) and (4,3) *e*) (–2,2) and (10,2) *f*) (–1,–1) and (–1,3)

- Determine the equation and the intercepts (and do the graph) of the straight line passing through the point (1,1), which satisfies the additional condition:

  a) the x-intercept is $x_0 = 5$ *b*) the y-intercept is $b = -3$

  c) it is parallel to the line $3x-2y = 4$ *d*) it is perpendicular to the line $x+2y = 4$

- Find the coordinates of the intersection point of the two straight lines and make a graph including both lines and all the points involved:

  a) $3x-2y = 4$ and $x+2y = 4$ *b*) $x+y = 2$ and $x-y = -1$ *c*) $2x+y = 0$ and $4x+2y = -1$

1.3. Equations of parabolas – quadratic (2^{nd} degree) functions

Normal equation: $y = a\,x^2 + b\,x + c$, where a = main coefficient ($\neq 0$), $c = y$–intercept

Peak-point equation: $y = a\,(x - x_0)^2 + y_0$, where $x_0 = -\dfrac{b}{2a}$ and $y_0 = c - \dfrac{b^2}{4a}$ are the coordinates of the *peak–point* of the parabola (obtained from normal equation by *completing the square*)

Factorized equation: $y = a\,(x - x_1)\,(x - x_2)$, where x_1 and x_2 are the x-intercepts, that is, the roots (zeros) of the quadratic function: $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

PROBLEM 3 EQUATIONS OF PARABOLAS – QUADRATIC (2^{ND} DEGREE) FUNCTIONS

- Determine the peak-point equation and (if exists) the factorized equation of the parabola given by the following equation. Also give its notable points (x- and y-intercepts, peak-point), and make a graph!

  a) $y = -x^2 - 5$ *b*) $y = 10x^2 - 50x$ *c*) $y = x^2 - 4x + 3$ *d*) $y = 2x^2 - 4x + 2$

  e) $y = 2x^2 - 10$ *f*) $y = -x^2 + x - 1$ *g*) $y = -0.5x^2 + x - 2$ *h*) $y = -2x^2 + 7.5x + 2$

Note: to make a graph is always a must! The graph should always contain all geometric objects (point, straight line, parabola) involved in the problem. In case of sketching the graph of a function all notable points should be marked. Along the x- and y-axes of the coordinate system, the scale (often integer numbers) must be given detailed enough to cover the range of the graphed objects.

PROBLEM 4 ELSE: GRAPHING THE ABSOLUTE VALUE (|*x*|), SIGN (*SIGNUM*) AND RECIPROCAL (HIPERBOLA) FUNCTIONS

a) $y = |x| \equiv \begin{cases} -x, & \text{if } x<0 \\ 0, & \text{if } x=0 \\ x, & \text{if } x>0 \end{cases}$ b) $y = \operatorname{sgn} x \equiv \begin{cases} -1, & \text{if } x<0 \\ 0, & \text{if } x=0 \\ 1, & \text{if } x>0 \end{cases}$ c) $y = 1/x$

PROBLEM 5 GRAPHICAL EXERCIES

Which curve (find such pieces!) represents a graph of a function, or one-to-one function?

(1)
(2)
(3)
(4)
(5)