Non-Standard Forms of Teaching Mathematics and Physics: Experimental and Modeling Approach

IPA HU-SRB/1203/221/024

Visualized Problems in the Teaching Topic "Derivative of a Function"

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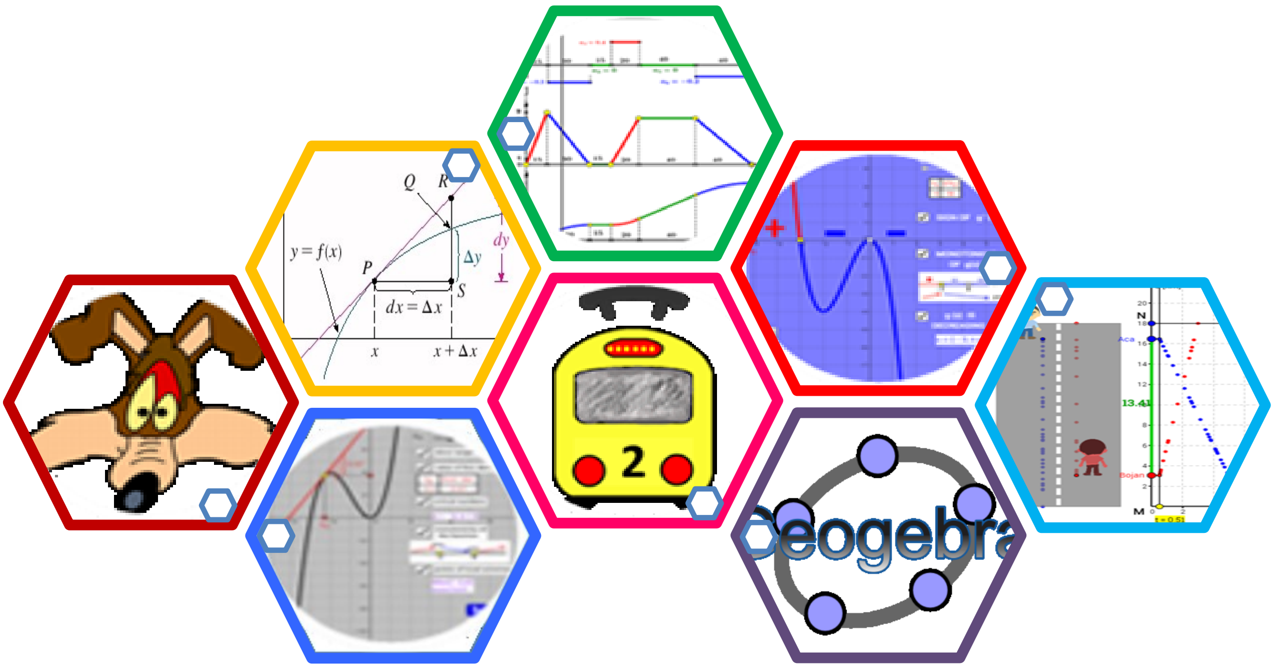

Novi Sad 2015

The project is co-financed by the European Union

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Preface

Some possibilities for implementing cognitive-visual approach in calculus are visualized problems. These are problems in which the image is explicitly or implicitly included in the very manner the problem is formulated, in the way of solving the problem or in the final solution. We are presenting examples of how GeoGebra is used in classrooms, to explain and explore concept of first derivative and its properties and applications. The concept of local extreme values of functions is also one of the themes in this teaching material. The basic idea while preparing GeoGebra dynamic worksheets is connecting algebraic and geometrical interpretation of the concept of derivative of a function.

Uniform motion and motion problems are teaching topics which are taught in physics class in school. The problems of uniform movement are an integral part of the teaching contents of mathematics, both in primary as well as secondary school. Mathematical and physical approach to the solution of specific tasks may vary. In the second part of teaching materials we present how GeoGebra can be used in modeling the problem of uniform movement and movement problems and how the created models can be applied in the classroom. We prepared two simulations, one which refers to uniform motion and the other refers to motion problems realised with application of derivations and integrals.

1. Visualized Problems in the Teaching Topic "Derivative of a Function"

Some possibilities for implementing cognitive-visual approach in calculus are visualized problems. These are problems in which the image is explicitly or implicitly included in the very manner the problem is formulated, in the way of solving the problem or in the final solution. Presented GeoGebra material shows examples of such problems. Function graph (Derivative's graph) is displayed in the grid, so that the image contains the data required for the solution. If the figure shows a function graph, then the first derivative is discussed and vice versa. To solve the problem, it is necessary both to apply the visual understanding and thinking and the knowledge of various mathematical fields.

We are presenting examples of how GeoGebra is used in classrooms, to explain and explore concept of first derivative. The basic idea while preparing GeoGebra dynamic worksheets is connecting algebraic and geometrical interpretation of the concept of derivative of a function.

Problems with graphs of first derivatives

Functions f(x), g(x) and h(x) are defined and continuous on the set R, while the function s(x) on a set R\{0}. The figures show graphs of their first derivative. Based on these graphic solve the following problems:

PROBLEM	CHECK YOUR SOLUTION	
1. The function g(x) is decreasing ⇔ x ∈	☐	Help
2. The function s(x) has a local maximum at x =	☐	Help
3. The function h(x) has an global minimum on the interval [0, 4] if x =	☐	Help
4. Tangent line t to the graph of f(x), at point (x_0, y_0), is parallel to the line l: y = - 4x+3. Then it is x_0 =	☐	Help
5. Determine the x-coordinates of the points on the curve y=h(x), for which the tangent makes an angle of 135° with the x-axis.	☐	Help

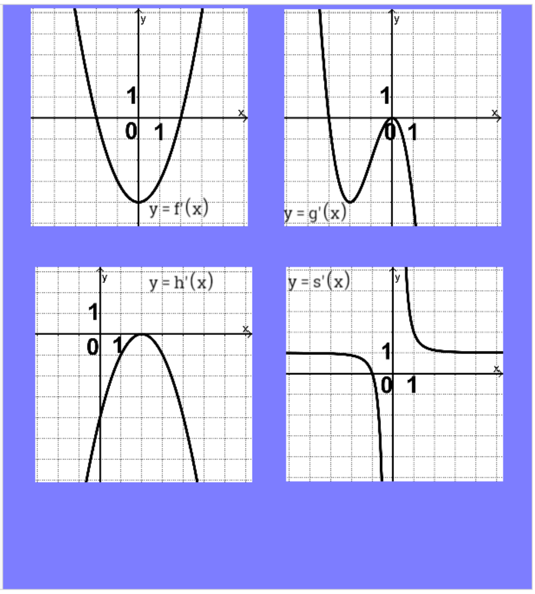

Figure 1.1

Dynamic worksheets allow students to verify if they solved the problem correctly (check your solution). In case that students haven't solved the assignment correctly, or if during solving they have certain difficulties and concerns, they can use additional explanations by clicking the "Help" button. The assistance that students can receive is organized in multiple levels, and is shown by checking the corresponding boxes.

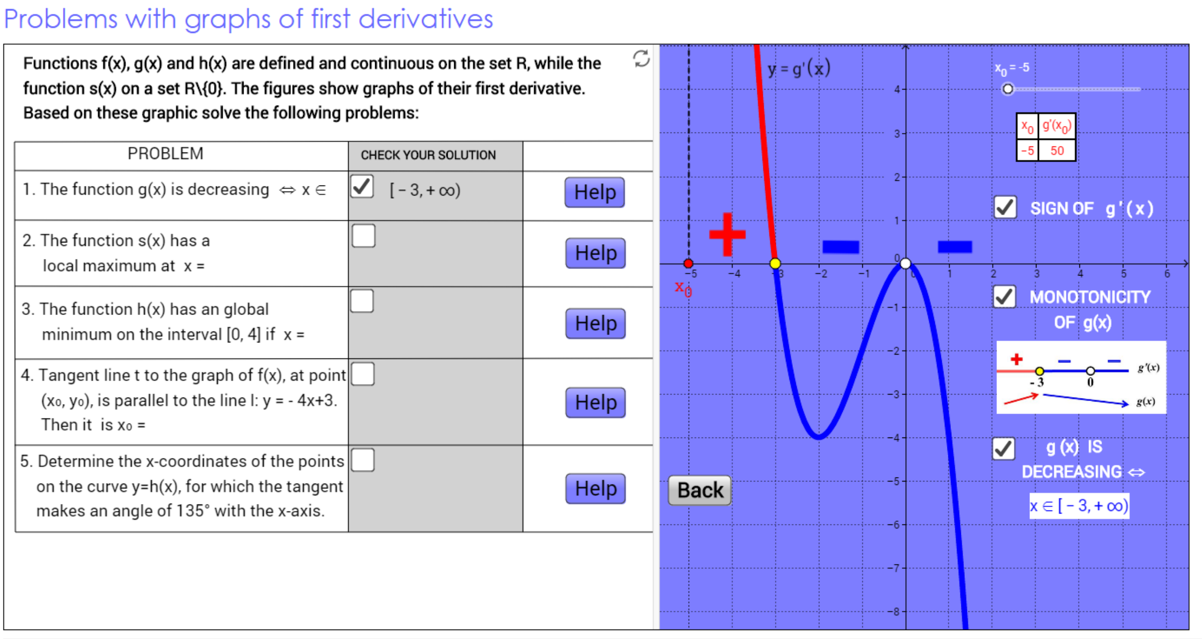

Figure 1.2

GGB Tube: http://tube.geogebra.org/material/show/id/1120005

2. Functions' Monotonicity, Extreme Values and Derivative

The story is based on the famous cartoon where the main character, Willie E. Coyote is trying to catch Road Runner. He is crossing the wire holding the anvil in his hands. Due to the weight of the anvil, the wire stretches, lowering the Coyote on the ground. He at that point drops the anvil, and since he is on the wire, he was launched into the air. For a period of time he is in the air flying up and then reaches a maximum height and begins to fall down. Ultimately, he falls to the ground.

With the help of GeoGebra we created one simulation, which consisted of three elements:

- The first element represents the function graph and its derivation. The animation shows the path of the cartoon hero, represented mathematicaly by function graph. This is a very effective and obvious way to associate the real situation with the mathematical content.
- Second element of GeoGebra animation is used for representing changing of function values. Specifically, this element of animation shows how function changes it's values, by using image of cartoon hero moving up and down in line with the changing value of the function. This section was inserted in an animation to illustrate the increase or decrease in function values. We used this part of the animation to explain students the connection between real situation(coyote is moving), with mathematics(function graph).

- The third section shows the sign of the functions first derivative. All three sections are connected by the same variable, so that at any time students can connect position of the cartoon hero with function monotonicity and with the sign of the functions first derivative. It was deliberately done in this way, in order to point out to the students the connection between functions first derivative and it's monotonicity.

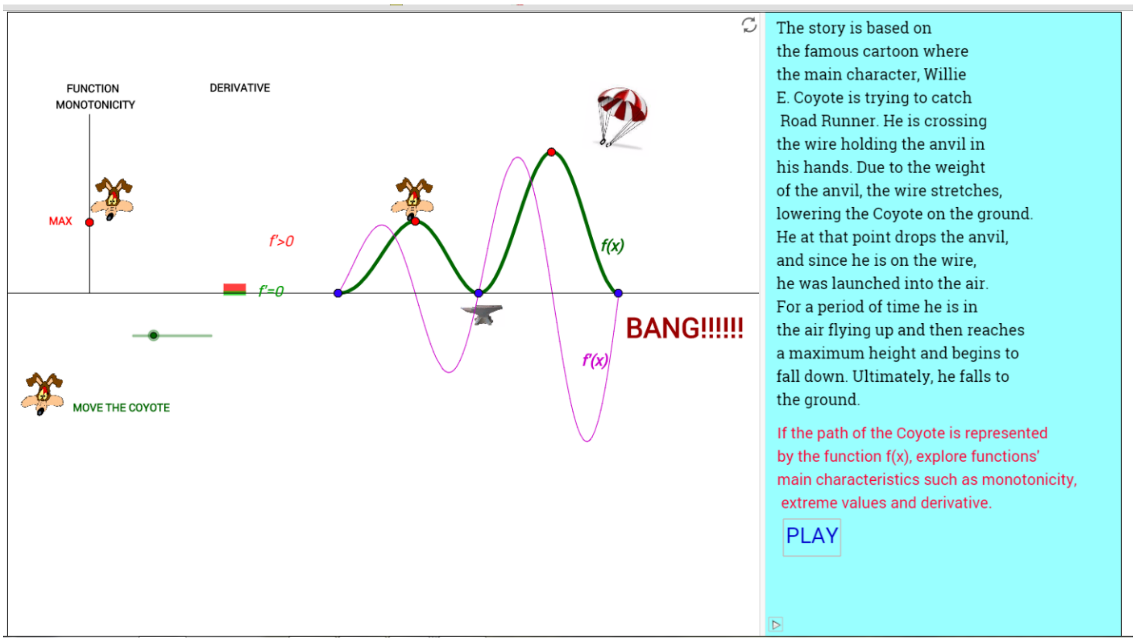

Figure 2.1

Specifically, the animation not only reflects the real situation on the mathematical model, but it can be manipulated, which is the primary purpose of mathematical modeling – to make model whose parameters can be changed, in order to simulate changes in real situations.

Without the use of computers in mathematical modeling process it would not be possible to manipulate with the model and especially to perceive the real situation. In particular, by using GeoGebra is possible to "launch" the model. In this way the maximum effects are achieved in terms of understanding the concepts that students have encountered in the process of modeling.

Thus, the GeoGebra can be used for representation of function graph, and there are also options allowing to display functions first derivative as was used in the first segment of the animation of the mathematical model.

GeoGebra's other options that were used in this study are related to inserting images and text. Images in GeoGebra can be further animated, and it was used in the study for animation of the cartoon hero picture, resulting painting the whole situation with mathematical terminology.

The key point in this study has found an GeoGebra option called slider. This option gives the user the ability to set the interval and the steps in this interval by which the value of a variable is taken. It is possible to launch slider and animate various structures in GeoGebra, and that is the one of GeoGebra's main advantages as seen from the mathematical modeling point of view.

The changes are caused by moving the slider in the bottom left corner of the screen. So, when the first derivative is positive, the function is increasing and when the function first derivative is negative function is decreasing.In this case, the slider is present in each section of the GeoGebra animation. In the first section slider is used to run the cartoon character walking on the function graph. In the second section slider starts cartoon character image that goes up and down, and illustrates the rises and falls of the function values. Finally, in the third section, slider creates polygons depending on the sign of the function first derivative, and by that draws a connection between the sign of the function first derivative and its monotonicity.

GGB Tube: http://tube.geogebra.org/material/show/id/1119673

3. Extreme Values of Function in Geogebra Style

Dynamic worksheets which we are presenting were prepared and used in classroom with students of the fourth grade of high school. The basic idea while preparing GeoGebra dynamic worksheets was connecting algebraic and geometrical interpretation of the concept of local extreme values of functions. The problem which is given to the students was as it follows: link each of four given functions with its derivative function. Functions are given with their graphs, and their derivative functions are given analytically, namely with formula.

Before solving the problem, the following is to be repeated with students: the notion of the critical number (critical point) of the function, the notion of the local maximum and minimum of the function, necessary and sufficient conditions for local extreme points and procedure for determining local extremes of the function. Students on the basis of the graph of each function, get essential data of critical numbers and local extremes of the function. As the derivative functions are given by formula, by conducting the corresponding algebraic procedure, students first determine the critical numbers, and then the points of local extremes. For the students to link the function with its derivative function, they first need to connect the facts that they have learned by graphical and algebraic solving of the problem.

Students can check if they solved the problem correctly (check your solution), as well solving the problem again ("reset" button). In a case that students haven’t solved the assignment correctly, or if during solving they have certain difficulties and concerns, they can use additional explanations by clicking the "help" button.

Puzzle-graphs functions and their first derivatives

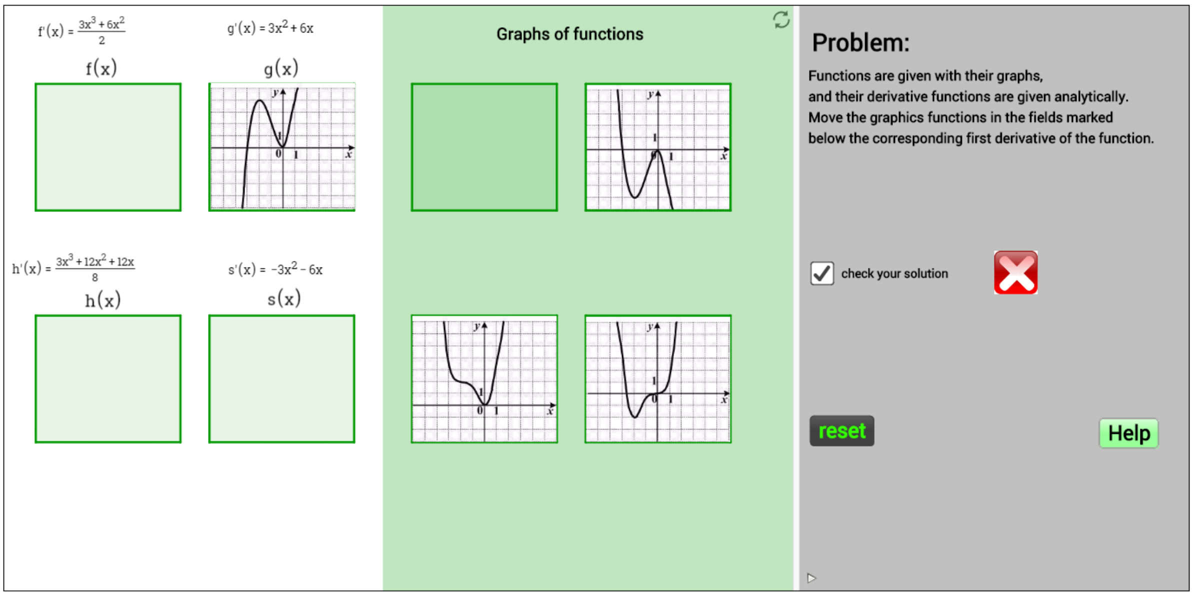

Figure 3.1

The assistance that students can receive is organized in multiple levels, and is shown by checking the corresponding boxes or clicking on the button.

The levels of help for derivative functions are:

1) Real numbers for which the first derivative of the function is not defined.

2) Zeroes of the first derivative.

3) Critical numbers of the function.

4) Sign of the first derivative and monotonicity of the function.

5) Local extremes of the function.

Assistance with graphs of the functions is given in following levels:

1) Tangents of the graph of the function and value of the first derivative - Students can move the slider and observe the movement of the point on the function curve and the tangent line which is constructed at that point. The slope of the tangent line is geometrically represented by the corresponding right-angled triangle. Also by moving the slider, students can follow the changes in the value of the functions’ first derivative.

2) Critical numbers of the function.

3) Monotonicity of the function.

4) Points of local extremes.

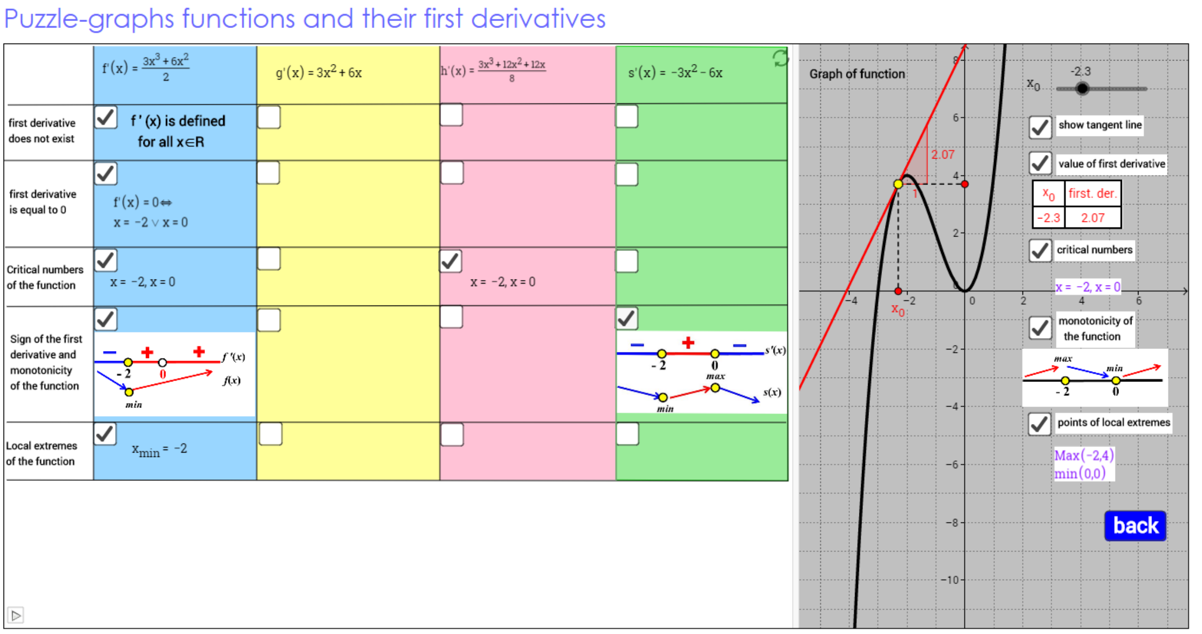

Figure 3.2

According to their needs, students can choose any type of help, and with independent research they can come to certain conclusions and knowledge. After using selected levels of assistance, by clicking the "back" button, students have the possibility to return to solving the problem, and checking the solution.

GGB Tube: http://tube.geogebra.org/material/show/id/1119989

4. The Mathematical Modeling in Practice - Problems of Uniform Movement Modeled in Geogebra

Uniform motion is teaching topic that is taught in physics class in the 6th grade elementary school and in the first year of high school. The problems of uniform movement are an integral part of the teaching contents of mathematics, both in primary as well as secondary school. Mathematical and physical approach to the solution of specific tasks may vary. We present how GeoGebra can be used in modeling the problem of uniform movement and how the created model can be applied in the classroom

Problem: Locations M and N are located at a distance $d = 18\ km$. Bojan is traveling from place M to place N, velocity $v_1 = 6\ km/h$. Aca travels from place N to place M, velocity $v_2 = 3\ km/h$. If they start at the same time,

1) After how many hours will they met?

2) After what time they were at a distance $d_1 = 4\ km$?

3) At what distance they were after $t_1 = 2{,}5h$?

This problem was modeled graphicaly, using linear functions, because the distance from point M is in a linear dependence on time. Time was applied onto x-axis, and the distance from point M onto the y-axis. Aca's distance from point M is given by function $y = 6x,\quad 0 \le y \le 18$, and Bojan's distance from point M by function $y = 18 - 3x,\quad 0 \le y \le 18$.

Dynamic worksheet created in GeoGebra was used to model a given problem and where the simulation of Aca's and Bojan's movement is also possible. Moving the slider for the time t, figures of Aca and Bojan are moving with the appropriate speed and in the coordinate plane appears trace of plotted graphs of their distance from the point M, depending on the time. At the same time in the appropriate fields we can see the change of routes they crossed length, and the length of the distance between them. At a time when their distance is equal to zero, ie. when Aca's and Bojan's are figure next to each other, students can read the time of their encounter.

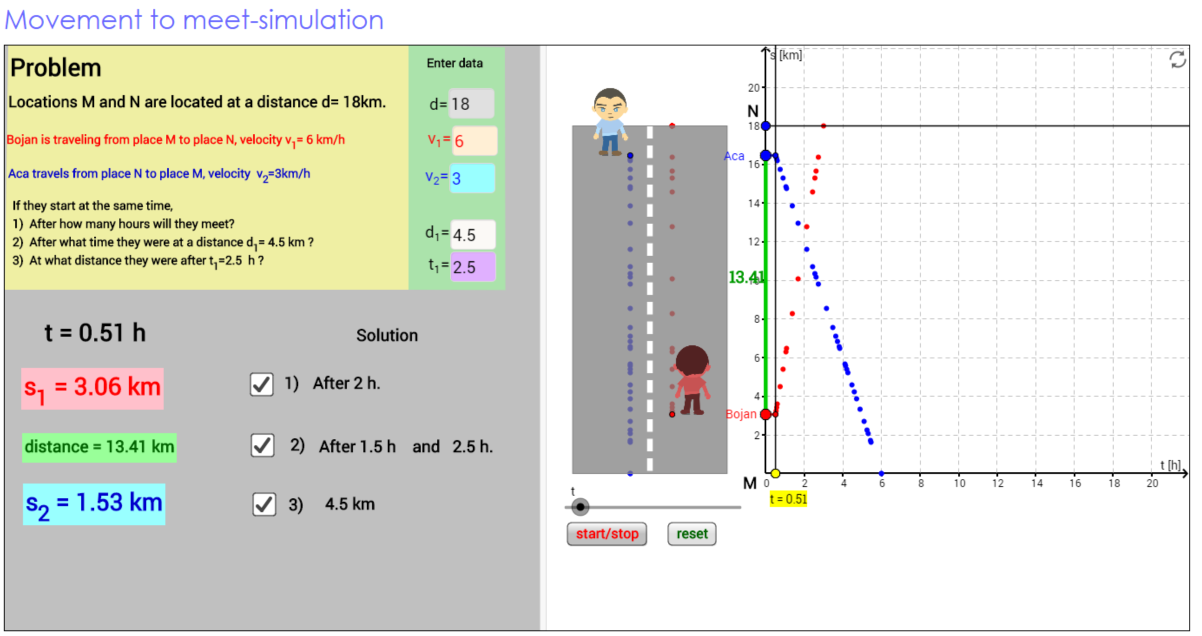

Figure 4.1

We can continue to move the slider until the moving of figures is finished. In the coordinate plane are shown the graphs of the distance functions from point M, for Aca and Bojan. By checking the appropriate box, the solution can be checked.

In this dynamic worksheet, it is possible to change the input data, so the students are able to independently investigate the problem of movement of two bodies. After linking the essential visual features, students can more successfully approach to solving the task using algebraic method, symbolic representation and logical-deductive thinking.

GGB Tube: http://tube.geogebra.org/material/show/id/1119963

5. Movement Problems in Teaching Practice

The motion problem was realized in GeoGebra. The GeoGebra material consists out of five sections, each separately constructed for illustration of specific part of the presented problem. Each section is available from other sections by checking its box in upper left corner of the GeoGebra window.

Section I

Section I, marked with ***Problem***, consists out of two parts, called Graphics and Graphics 2 in GeoGebra. Graphic 2, on the left part of the screen gives detailed textual description of the motion problem. Right part of the screen is graphical representation of the considered problem.

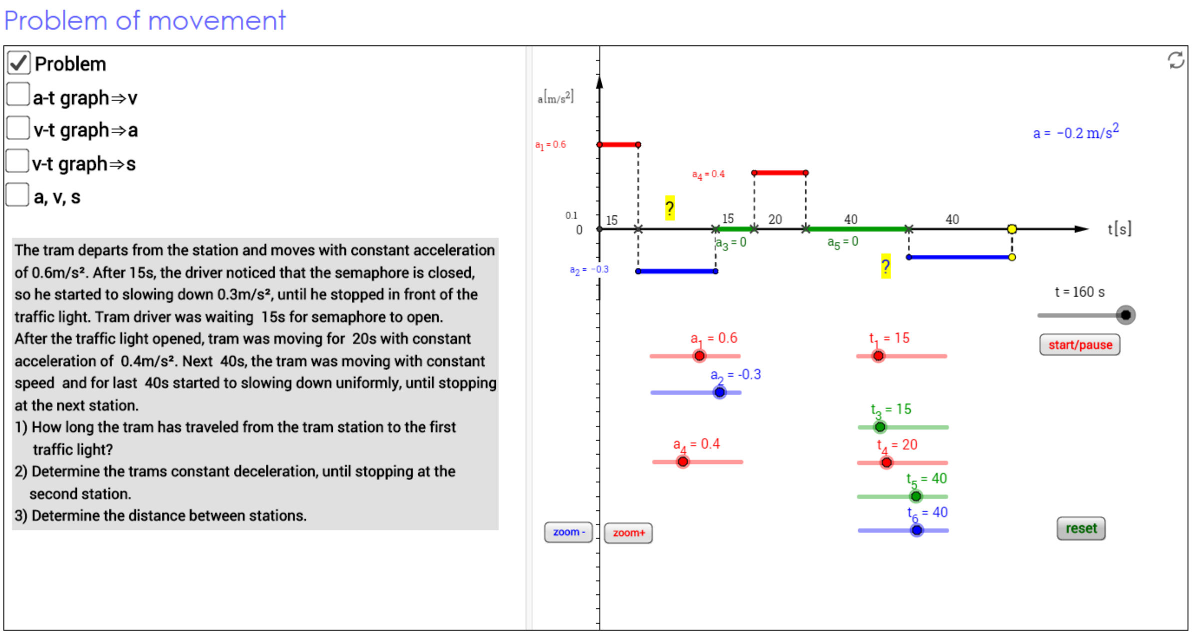

Figure 5.1

The bottom part of right side of the window, includes so called *sliders*, a special GeoGebra tools, which allow users to set an interval for values and the steps in this interval by which the value of a variable is taken. In that way, GeoGebra gives users the options to change parameters values, and by that to make simulations and animations. In the movement problem, sliders are used for interval representation of problem parameters (velocity, acceleration, deceleration, time, distance,...). Also, the user has the opportunity to change input parameters, by moving the sliders, and, in that way, to create diferent movement problems and situations.

Section II

This section appears when checking the box ***a-t graph →v*** in upper left corner of any window of GeoGebra simulation. Section II consists out of two graphic, and its aim is to connect acceleration with velocity by calculating velocity using for parameters acceleration and time. Left part of the window shows tables with data for time and acceleration, and velocity, when calculated. Here, for calculation of velocity, we use the following formula:

$$v = a \cdot t$$

(v-velocity, a-acceleration, t-time)

Right side of the window gives graphical representation of acceleration and velocity, and also by clicking *start/pause* button it is possible to start simulation of whole process of calculating velocity, when acceleration and time is changing.

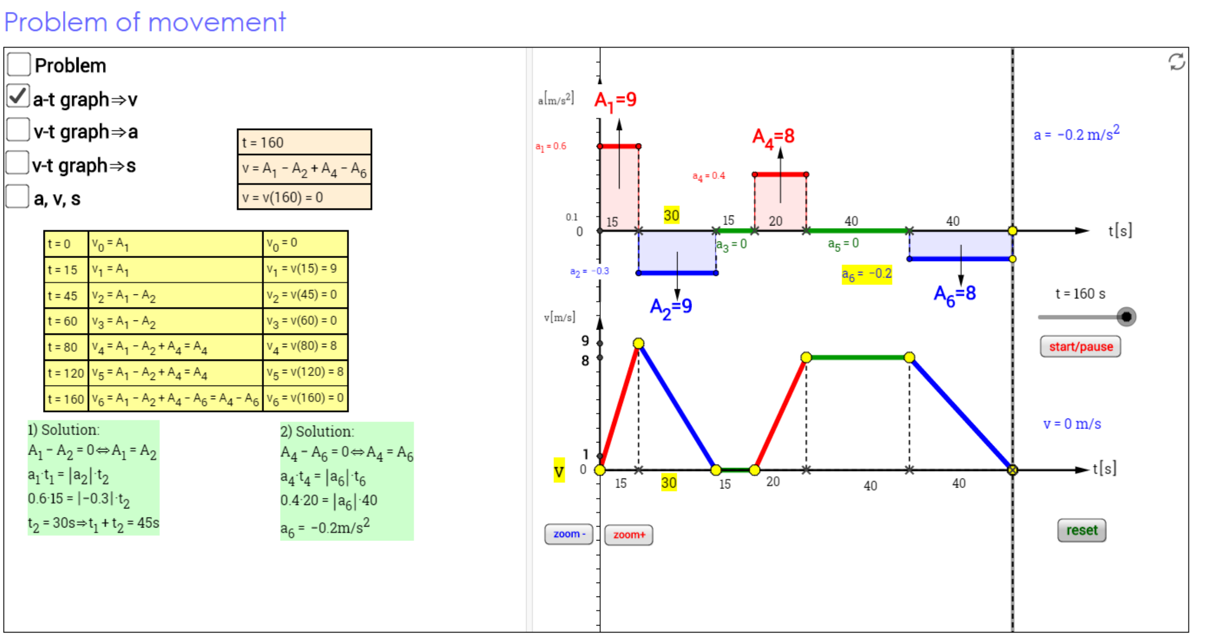

Figure 5.2

It is very interesting to mention the method of interpretation of calculating process, because this method was used intentionally to illustrate the connection between acceleration, time and velocity.

Acceleration graph has acceleration on its ordinate and time on its abscissa, upper right corner of Section II. By calculating velocity, we actually calculate area of rectangles appearing on acceleration graph. These rectangles have time parameter representing its one side, and acceleration parameter representing its other side. The lower graph represents velocity, and its direct connection with acceleration. This way of velocity calculation, using rectangles areas gives students one more opportunity for visualization of presented problem and its parameters.

Section III

Section III illustrates acceleration, its calculation and connection with velocity and time. When user check the box marked with ***v-t graph →a***, it automatically follows to the Section III window. This section window is also divided in two parts. One, on the left side, presents parameters data in a table and on the right side change of parameters is showed as simulation. The leading idea for this kind of representation is to connect data and their mathematical representation with visual simulation. In that way we indicate at the main point of mathematical modeling and its application in mathematics education and beyond.

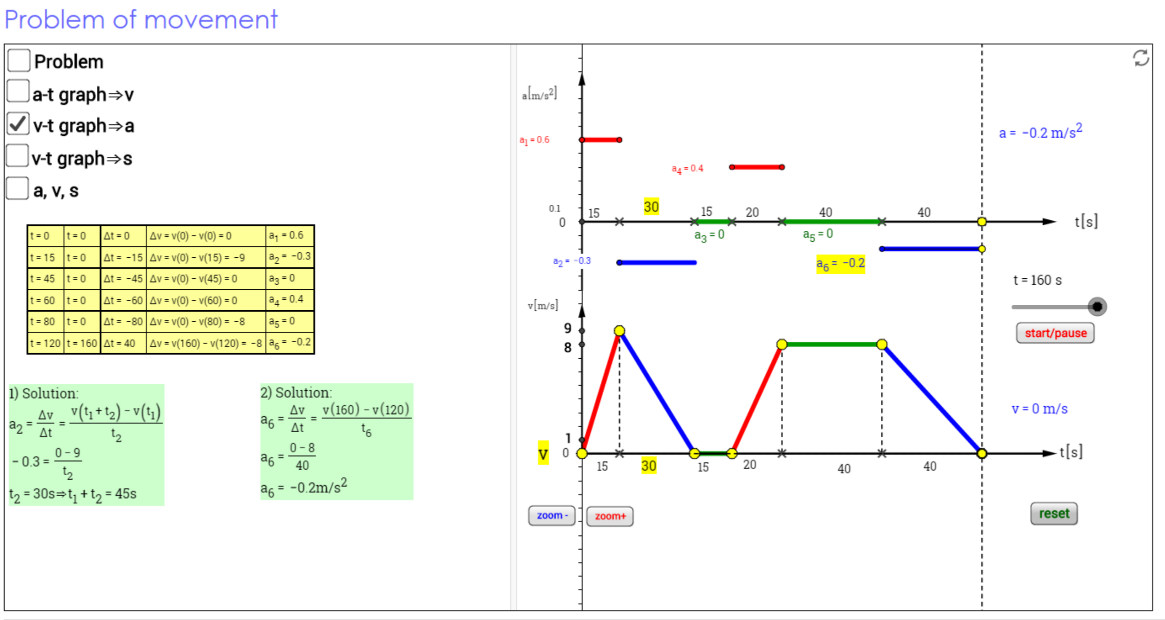

Figure 5.3

In order to involve different mathematical theories in this model, acceleration is calculated as change of velocity over change of time:

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta \boldsymbol{t}}$$

(a-acceleration, Δv*-change of velocity,* Δt *–change of time)*

Calculating acceleration in this way, we added one more dimension in mathematical model of movement problem – derivative. So, this mathematical model can be used for teaching not only movement problems, but also derivative and its applications on real world problems.

Section IV

Section IV illustrates solution of part of the movement problem concerning the covered distance.

Using velocity and time as parameters, distance was calculated. Right side of the Section IV window is graphical representation of distance and its calculation mode. Upper graph on the right side of Section IV shows velocity as function of time. Since velocity is defined as change of distance over change of time, meaning velocity is derivative of distance over time:

$$\boldsymbol{v} = \frac{\Delta \boldsymbol{s}}{\Delta \boldsymbol{t}}$$

(v-velocity, Δs-change of distance, Δt –change of time)

In order to calculate distance, we calculated it as area of triangles (rectangles) determined by velocity graph. Actually, distance was calculated as definite integral of velocity over time:

$$s = \int_{t_1}^{t_2} v(t)\, dt$$

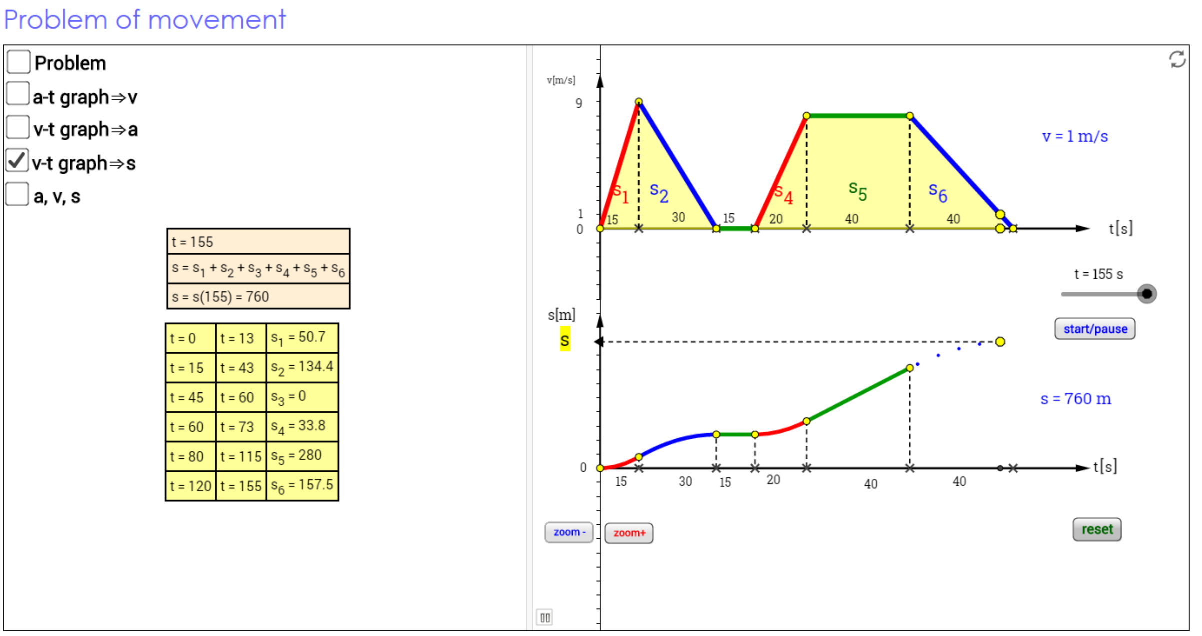

Figure 5.4

In that way, integrals are also part of mathematical model used, which broadens the scope of its application.

Section V

Section V unites all four previous sections, and through graphical simulation and animation illustrates the presented problem. Right side of the window is graphical simulation of acceleration, velocity and distance, and by their simultaneous change, students can observe dependency of these three concepts. Also, from graphical simulation can be noticed that different methods and theories can be used for calculation of acceleration, velocity and distance, and that all of them interact with one another.

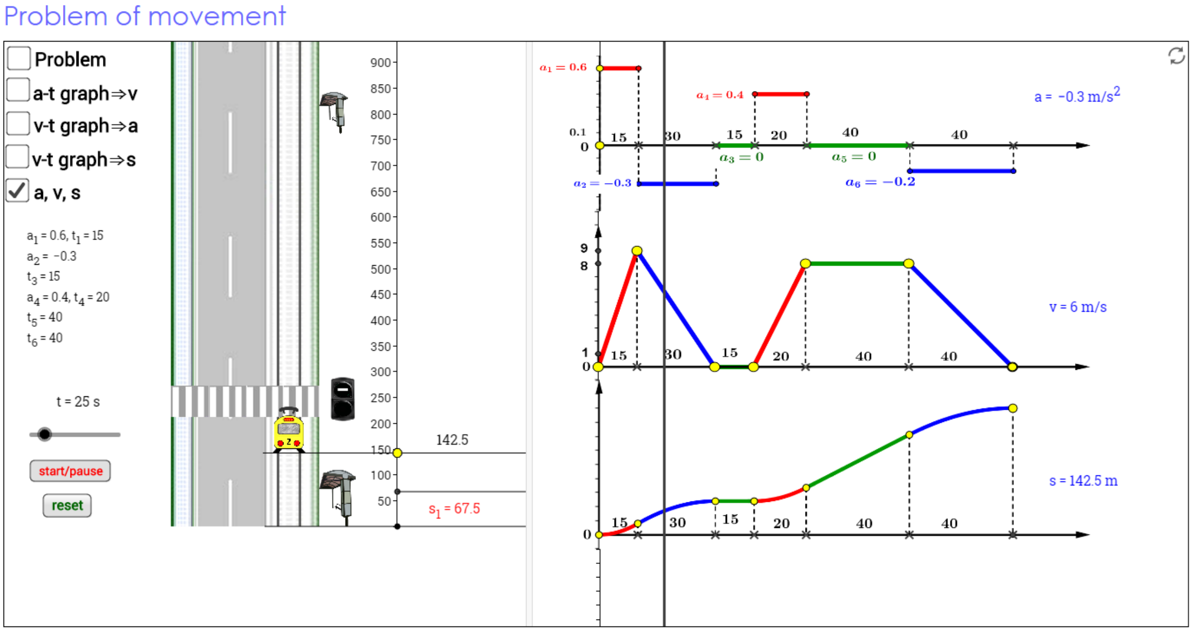

Figure 5.5

For the purpose of realization of motion problem simulation we used area of triangle and rectangle, derivative and definite integral. Using different theories for describing mathematical model of motion problem, the authors wanted to achieve the possibility of implementing this mathematical model on different levels of mathematics education. So, when using this model in elementary school, it can be described with areas. Approach with derivatives and definite integrals could be used in higher education, and also at university level.

GGB Tube: http://tube.geogebra.org/material/show/id/527765