Spring School on Mathematics and Computer-Aided Modeling in Sciences

Szeged - Novi Sad

2011

organized by

University of Szeged

and

Faculty of Sciences, University of Novi Sad

in the frame of the IPA Cross-border Co-operation Programme

HU-SRB/0901/221/088

"Teaching Mathematics and Statistics in Sciences: modeling and computer-aided approach"

A program a Magyarország – Szerbia IPA Határon Átnyúló Együttműködési Programban az Európai Unió társfinanszírozásával valósul meg.
Projekat sufinansira Evropska Unija u sklopu IPA prekogranicnog programa Mađarska-Srbija
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School summary

Dates:

- Szeged: 05.19.2011 - 05.21.2011
- Novi Sad: 05.27.2011 - 05.29.2011

Groups:

- PhD students, researchers (In English)
- high school teachers, students (Szeged: in Hungarian, Novi Sad: in Serbian)

Audience:

We welcome PhD students, researchers, high school teachers and academics from both sides of the border who work in mathematics, physics or other sciences. Talented undergraduate students may also apply with the recommendation of their supervisors.

Participating conditions:

- Participation is free, participants will receive course materials on DVD.
- We have limited possibilities to support accommodation in student hostels. Participants from the other side of the border and participants invited by their results on competitions are of high priority. Travel costs are covered by the participants.
- We ask the participants to give seminar at their workplaces or schools for their colleagues and/or students from the subjects they studied on the courses. A report should be prepared from this presentation, illustrated with photographs, which will appear on the website of the project.

Information, WWW:

- Szeged: www.model.u-szeged.hu
- Novi Sad: http://sites.dmi.rs/projects/IPA/

Contact

Szeged:

- János Karsai PhD, Associate Professor, project manager, karsai@dmi.u-szeged.hu
- Eszter K. Horváth PhD, Assistant Professor, coordinator, horeszt@math.u-szeged.hu

Novi Sad:

- Arpad Takači PhD, Professor, project manager, takaci@dmi.uns.ac.rs

Programme of the schools

The courses will concern interesting applications, modeling problems and tools in several areas of sciences in such a way that it will be enjoyable also for non-mathematicians as well.

The participants will learn some parts of the following fields: computer-based mathematical applications; computer-aided studies of physical, and biological-chemical models, geometrical structures; digital geography, statistical data analysis; signal processing and computer-aided measuring techniques. For participating teachers, we emphasize the didactic aspects of these techniques. The schools in Szeged and Novi Sad are independent, but they well complement each other. The courses will be held in computer rooms. The participants will study the topics via practical examples.

Acreditation: The school is accredited by the Mathematics Doctoral School at University of Szeged (5 credits). It is also accredited as a teachers' training in Serbia.

The programme of the School in Szeged: (May 19-21)

Lecturer	Course	Hours
Plenary talks		
Tibor Krisztin, professor, Bolyai Institute	Can we understand the unpredictable? The mathematics of chaos	1
Péter Maróti, professor, Department of Med. Physics and Med. Informatics	Simulation in Biology	1
Talks of some prize-winners on the high school competition (2010):		1
Eliza Bánhegyi József Attila Gimnázium, Makó Teacher: Erika Rójáné Oláh	Then and Now - Triangulation	
Andor Viharos Radnóti Miklós Kísérleti Gimnázium, Szeged Teachers: Gábor Ábrahám, Kosztolányiné Erzsébet Nagy, Lajos Pósa, István Tigyi	Thoughts about the game Maffia	
Bernadett Juhász-Bóka Horváth Mihály Gimnázium, Szentes Teacher: Volosin Vlagyimir	Solving quadratic equations by geometric ways	
Annamária Kiss Arany János főgimnázium, Nagyszalonta Teacher: Julianna Mészár	Ramanujan	
Group of researchers and PhD. students (English)		
Danijela Rajter-Ćirić, professor, Univ. Novi Sad	On the probability and stochastic models	2
János Karsai, associate professor	Study of dynamic systems with Mathematica	4
Zoltán Kovács, assistant professor	Fractal design with computers	3
Ferenc Peták PhD, associate professor, Dorottya Czövek PhD student	Medical physics practical	4
Eszter Katonáné Horváth, assistant professor	Islands	2
Róbert Vajda, assistant professor	Symbolic and numerical study of equations and inequalities with computer algebra software	*4*
Consultation, workshop		2x1
Total hours:		**3+21**
Teachers, didactic PhD. students, high school students (Hungarian-English)		
Đurđica Takači, professor **Univ. Novi Sad**	Mathematical modeling with Pascal triangle (English)	2
Lajos Szilassi, associate professor	The camera, what is in front and behind (The specialties of axonometric and perspective visualization)	2
László Égerházi, assistant professor, Dorottya Czövek PhD student	Medical physics practical	4

Krisztina Boda, associate professor	Basic biostatistics with applications	3
Géza Makay, associate professor	Building a conference center or triangulating surfaces	2
Zoltán Gingl, professor Péter Makra, assistant professor	Demonstration experiments with a sound card	2
Attila Máder, assistant professor	Computer-aided, exploration centered teaching of math in the practice II. Experimental Mathematics in Action in the Classroom	2
Zoltán Kovács, assistant professor	Free software in mathematics education	2
Consultation, workshop		2x1
Total hours:		**3+21**

The programme of the School in Novi Sad (May 27-29)

Lecturer	Course	Hours
Group of researchers and PhD. students (English)		
Arpad Takači, professor	Introduction to Traffic Flow Theory	2
János Karsai, associate professor	Some applications of impulsive systems	2
Krisztina Boda, associate professor, Univ. Szeged	Some experiences using multivariate modeling methods en paediatric aneasthesia	2
Dušan Mijatović, assist. prof.	Mathematica and Anylogic Workshop in Computer laboratory	4
Stevan Pilipović, academician	On the visualizations of generalized functions I	2
Marko Nedeljkov, professor	Some physical examples	2
Danijela Rajter-Ćirić, professor	On the probability and statistical models	2
Đurđica Takači, professor	On the visualizations of generalized functions II	2
Branimir Šešelja, professor	Fuzzy sets and logic with applications in natural sciences I	2
Andreja Tepavčević, professor	Fuzzy sets and logic with applications in natural sciences II	2
Srđan Škrbić, assistant professor	Mathematical background of relational data model	2
Total hours		**24**
Teachers, didactic PhD. students, high school students (Serbian - English)		
Arpad Takači, PhD, professor	Mathematical modeling and simulation in teaching mathematics	2
Géza Makay, associate professor, Univ. Szeged,	Sudoku: methods of solutions, how to generate and solve examples with computer; Sudoku competition	2
Attila Máder, assistant professor, Univ. Szeged	Computer-aided, exploration centered teaching of math in the practice II.	2
Đurđica Takači, professor	On the definition of the logarithmic function, mathematical modeling of earthquake	4
Teodor Atanacković, PhD, academician	Introduction to fractional calculus with the applications	2
Marko Nedeljkov, PhD, professor	Mathematical models from physics, with the accent on the high school contents	2
Duška Pešić, PhD, high school teacher	Visualization of recursive sequences with Geogebra	2

Šešelja Branimir PhD, professor	A generalization of characteristic, fuzzy sets and fuzzy relations, their basics properties	2
Andreja Tepavčević, PhD, professor	Applications of fuzzy sets and fuzzy relations in contemporary technology (fuzzy controllers in washing machines, video cameras, etc.)	2
Srđan Škrbić, PhD, associate professor	Data basis approach in high school	2
Danijela Rajter Ćirić, PhD, professor	Didactical approach of the teaching statistic in high school	2
Total hours		**24**

Summary of the lectures and courses of the school in Szeged

Plenary talks

Can we understand the unpredictable? The mathematics of chaos

Tibor Krisztin DSc., Professor of Mathematics
Bolyai Institute, University of Szeged, Hungary

The speciality of the chaotic dynamic systems is that the small change of initial data can result in a huge change in the development of the system in the time. Hence, the long time prediction of such systems is impossible. Nevertheless, there are mathematical tools, which can help to measure the phenomenon of the chaos. We show some of them via examples.

Simulation in Biology

Péter Maróti DSc., Professor of Biophysics
Department of Medical Physics and Medical Informatics, University of Szeged, Hungary

„If you give me two free parameters, I can describe an elephant.
If you give me three, I can make him wiggle his tail."
(Eugene Wigner (1902-1995), Nobel Laureate, Hungarian physicist)

The exact description of the biological processes needs a lot of freely chosen parameters. In general, so many that we could make the elephant even to dance. The scientist has the task and responsibility to select the appropriate form of dance.

The talk will offer you examples (and unsolved problems) of simulation in two distinct fields of the life sciences. Our aims are to demonstrate how 1) to set up models (to make abstraction), 2) to search for algorithms, 3) to construct elementary computer programs, 4) to compare the computational results with observations from laboratory and field experiments and 5) to draw the possible and realistic conclusions. The first topic focuses on special aspects of the ecology (population dynamics: logistic equation and the Lotka-Volterra treatment of competition and predator-prey-models). The second topic deals with an ongoing problem of the biophysics of molecules (the anticooperative interaction of protonatable groups within the protein and its significance in function of the biomolecules).

Based on the simulations, we know what *may control* the population (finite source of food (living-space or „Lebensraum"), cannibalism, competitive species, predators (parasitoids), diseases (pathogens), etc.), but we cannot say definitely what *actually does control* the population in the nature. We have two possibilities: 1) Observe the dynamics of the population *in vivo* und make model-calculations. It usually works fairly well but has a serious drawback: the method *discloses but does not proof* the possibilities. 2) Carry out field experiments (e.g. to eliminate the predators, to change the quantity of the food, etc.). The drawback of this method is the inherent limitation of some control mechanisms which cannot be either manipulated at all

(e.g. illnesses) or can be hardly modified in time (e.g. long living organisms as trees or tortoise) and/or in space.

The construction and calculation of models on molecular level are more comfortable as the results can be directly compared with the experiments. The drawback is usually the large number of freely running parameters. In majority of cases, we have to give up to determine the exact values of the fitting parameters and have to be satisfied with the results of simulation, which makes good feeling for the eye (inspection) but does not relax the volcanic mind (understanding) of the honest scientist.

Talks of some prize-winners on the high school competition (2010):

At that time and now - Triangulation

Eliza Bánhegyi

3. prize, interdisciplinary prize

Attila József High School, Makó

Teacher: Erika Rójáné Oláh

Triangulation is used in several areas of life. The talk will be given about the short history of its development, about its application in cartography, about the method. Moreover, the talk will contain the application of triangulation in practice and the importance of it in GPS system.

Thoughts about the Maffia game

Andor Viharos

3. prize

Miklós Radnóti High School, Szeged

Teachers: Gábor Ábrahám, Erzsébet Kosztolányiné Nagy, Lajos Pósa, István Tigyi

The talk will be about analysis of the well-known game, decisions during playing the game. Stochastic estimation on a simplified model will be presented. Finally, some words about a more complicated version of the game: what happens if we add a policeman to the game?

Solving quadratic equations by geometric way

Bernadett Juhász-Bóka

3. prize

Mihály Horváth High School, Szentes

Teacher: Vlagyimir Volosin

The talk will be given about one of the most comprehensive topics in high school mathematics; this time in an unusual way. Without the quadratic formula we draw the solutions only by compass and ruler, as it was done in ancient times.

Ramanujan

Annamária Kiss

Distinguished praise

János Arany High School, Nagyszalonta

Teacher: Julianna Mészár

The talk will be given about the unusual life and the activity of the ingenious mathematician, Srinivasa Ramanujan. During his 33 year life he produced more than 3000 formulas for the succeeding generations. Quoting Hardy, he had the most romantic personality among mathematicians, and by no doubt he was one of the most interesting individuals in mathematics.

Group of researchers and PhD. students

On the probability and stochastic models

Lecturer: Danijela Rajter-Ćirić, PhD, Associate Professor

Audience: Mathematics PhD students, researchers

Language: English

Planned length: 1.5 hours

Goal: Teaching and presentations of the probability theory and stochastic analysis methods, main notions and examples

Assumed knowledge: Calculus, basic Probability

Method: Presentation of stochastic processes through examples

Topics:

- Basics of probability theory
- Notion of stochastic process
- Some examples of stochastic processes

Tools:

- slides, computer, blackboard

Literature:

1. D. Rajter-Ćirić, *Probability* University of Novi Sad, 2009. (In Serbian)
2. S. Ross, *Introduction to Probability Models,* Academic Press, 2003.
3. D. Rajter-Ćirić, *Stochastic Analysis*, notes for students.

Study of dynamic systems with Mathematica

Lecturer: János Karsai PhD, Associate Professor

Audience: Mathematics PhD students, researchers

Language: English

Planned length: 4 hours in computer room

Goal: to give an introduction of the methods and tools of differential and difference equations in Mathematica via examples.

Assumed knowledge: Calculus of one and several variables, differential equations, computers

Method: Classes are held in computer rooms. The summaries are followed by solutions of practical problems. The teacher and the students work on the same problem simultaneously.

Topics:

- Basic tools of computer-algebra systems: use of *Mathematica*: numeric and symbolic calculations, complex arithmetics, variables, functions
- Elements of visualizations: functions, curves, surfaces, data, vector fields
- Differential equations in Mathematica: equilibria, vector fields, solutions, trajectories
- Qualitative methods: Liapunoff method, linearization
- iterations, difference equations

Tools:

- *Mathematica*

Notes, handouts:

- Electronic interactive projects, illustrations

Other literature

1. Karsai J., Impulzív modellek vizsgálata, Mathematica kísérletek, Typotex 2002
2. Karsai J., Mathematical programme packages, CD-ROM, 2008
3. Karsai J., Computer-aided mathematical modeling, CD-ROM, 2008

Creating fractals with the help of computer

Lecturer: Zoltán Kovács, Assistant Professor

Audience: PhD students in mathematics (researchers)

Language: Hungarian - English

Planned number of lectures: 3 hours in computer lab

Aim of course: The students will learn about mathematics software which are able to generate fast fractal animations. At the end of the course the students should be able to work with the applied software on their own, and obtain new information by utilizing a computer.

Prerequisities: functions in one and multivariable, differential and integral calculus, knowledge in using computers, the C programming language, basic knowledge of Linux

Method: The courses take place in a computer lab. Solution of practical problems will follow the summary of theory. Finally, the students will solve exercises on their own with the help of the lecturer.

Outline:

- Complex numbers, convergence and divergence on the set of complex numbers
- Classic fractals (Sierpinski carpet and triangle, Koch-curve), iterations
- Lindenmayer systems.
- Visualization of real and complex dynamical systems by using a spreadsheet software
- Installing XaoS from source code on Linux
- Creating a new fractal type in XaoS
- Installing and using RTZME (the Real-Time Zooming Math Engine)
- Analyzing and modification of fractal calculation methods in XaoS

Some topics for student homework:

- Creating Lindenmayer systems
- Programming Lindenmayer systems in C (for XaoS)
- Implementing fractal types from Fractint (for XaoS)

Applied tools:

- *XaoS, C development environment, internet, Linux, OpenOffice.org/LibreOffice Calc*

Bibliography:

1. J. Hubicka: XaoS 3.1.1. A fast real-time fractal zoomer --- hacker's guide, Feb 5, 2005
2. matek.hu knowledge base (in Hungarian) for XaoS
3. Kovács Z.: Véletlenszerű sorozatok, diszkrét dinamikus rendszerek, káosz (József Attila University, master thesis, 1999, in Hungarian)
4. Fractint documentation

Medical physics practical

Practice leader: Ferenc Peták PhD, associate professor, Dorottya Czövek PhD student

Audience: PhD students, researchers

Language: English

Planned length: 4 hours

Goal: To give an introduction to the signal processing in life sciences, sampling, filtering, analogue-to-digital conversion. Application of the knowledge for medical practice: electrocardiography (ECG), electromyography (EMG).

Assumed knowledge: Basic level from physics, biology and informatics

Method: After theoretical introduction, measurements by using Biopac system are performed followed by computer-assisted evaluations.

Preliminary programme:

- Introduction to the signal acquisition and processing, filtering, AD conversion
- Biophysical background of electrophysiological processes
- Measurement and evaluation of ECG and EMG

Tools:

- Biopac measurement system, computers

Notes, handouts:

- Lecture and practical presentations

Other literature

1. Damjanovich S., Fidy J., Szöllősi J. (Eds.) Medical Physics, Medicina, 2009

Islands

Lecturer: Eszter K. Horváth PhD, Assistant Professor

Audience: (Mathematics PhD students, researchers) or (high school teachers and students)

Language: English

Planned length: 2-4 hours

Goal: We define the notion of an island. We list the research results about islands. Followingly, we solve some exercises. Then we look for new problems, we discuss these problems, hopefully we partially solve the new problems.

Assumed knowledge: Induction, greatest integer function [x], elementary combinatorics. For the PhD course, abstract algebra and introductory lattice theory.

Method: We start with a lecture, mainly by using computer-projector, but also using blackboard. The participants solve some problems by themselves, with the help of the lecturer. Then, a brainstorming starts, as usual in mathematics research, i.e. we try to generate nontrivial but solvable questions. The participants might publish their results later.

Preliminary programme:

- The definition of an island, several grids.
- Czédli's formula for the maximum number of rectangular islands.
- Proving methods for the maximum number of islands.
- Triangular islands, square islands, estimates.
- The maximum number of islands on a cylinder, on a torus, on Mobius strip, in Boolean algebras.
- The maximum number of islands in case of finitely many height values.

Tools:

- beamer, computer
- blackboard, chalk
- swimming belt, swimming ball, paper cylinder, grid drawn on a paper.

Notes, handouts:

- Papers for the scientific background.
- Exercise sheet.
- Illustrations

Some topic for further individual investigation:

- Didactic investigation: Islands in students groups.
- Collecting similar or related exercises.
- New problems, new solutions.
- Visualizing islands, even by computer.

Equations and Inequalities with Computer Algebra

Lecturer: Róbert Vajda***,*** PhD, Assistant Professor

Audience: Mathematics PhD students, researchers

Language: English

Planned length: 4 hours

Assumed knowledge: high school algebra, computers

Summary: In mathematical models, we often search for objects whose components satisfy certain

constraints. Among the constraint satisfaction problems the simplest ones are the systems containing only equations and inequalities as constraints among the unknowns. We can be interested in satisfiability, i.e., for the existence of solutions, which might be dependent on certain parameters or we can search for the constructive description of the entire solution set. In this lecture we consider general algebraic systems over the real's.

First we consider the quantifier elimination problem over the real's and demonstrate the interplay among logic, algebra, and geometry. The problem of quantifier elimination can be solved by Collins' cylindrical

algebraic decomposition. Second, we consider the special case, if only equations occur among the constraints and introduce the basic notions of Grobner bases theory. The pure symbolic approaches have serious limits, because of the complexity of the underlying algorithms. Therefore, we also consider some numerical methods and built-in visualization tools at the end of the lecture.

Applied tools:

- *Mathematica*

Lecture notes, handouts:

- Electronic interactive notes
- Prepared handouts
- Electronic illustrations for the classroom work

Other literature

[1] Buchberger, Introduction to Gröbner Basis, 1998.
[2] Vajda, Effective Real Quantifer Elimination, Ebook Chapter, 2011.
[3] VajdaNumeric methods, electronic handouts, 2011.

Teachers, didactic PhD. students, high school students

Mathematical modeling with Pascal triangle

Lecturer: Đurđica Takači, PhD, Professor of Mathematics

Audience: Teachers

Language: English

Planned length: 2 hours

Goal: Teaching contribution in mathematical modeling as a tool in mathematical education.

Method: Visualization of the real problem and connection with Pascal triangle.

Preliminary programme:

- The construction of Pascal triangle;
- The different mathematics formula and Pascal triangle;
- Division of living cells, disintegration of the atom and Pascal triangle;
- Various real situations, as ordering food in the restaurant, coloring, … based on Pascal's triangle;
- Fibonacci sequence and curve, with GeoGebra.

Tools:

- beamer, computer
- *GeoGebra*

Notes, handouts:

- Prepared interactive projects in GeoGebra

Literature:

1. Dj.Takači, D. Pešic, Jelena Tatar *"On the mathematical modeling in teaching", Electronic book*

The camera, what is in front and behind
(The specialties of axonometric and perspective visualization)

Lecturer: Lajos Szilassi PhD, associate professor

Audience: High school students, teachers

Language: Hungarian

Planned length: 2 hours

Goal: The participants are introduced to the theoretical background of computer modeling of different situations in space geometry with special consideration to axonometric and perspective visualization. They get acquainted with some software for geometric modeling.

Assumed knowledge: High school geometry

Method: The classes are held in computer rooms, summarizing the theory is followed by solving practical problems.

Topics:

- Mathematical (and informatical) basis of preparation of visualization of 3D object.
- Different methods of axonometric and perspective visualization, with special concern of computer visualization.
- Comparison of the advantages and disadvantages of different axonometries.
- Use of dynamic geometry software in studying 2D and 3D geometry.
- Geometric contradictions by the Art of M C Escher.
- ***Tools:***
- *Dynamic Geometry software: Euklides, Geogebra Euler3D*

Literature:

- Lajos Szilassi: Geometry (manuscript in Hungarian)

Medical physics practical

Practice leader: Égerházi László PhD, assistant professor, Dorottya Czövek, PhD student

Audience: teachers, high school students

Language: Hungarian

Planned length: 4 hours

Goal: To give an introduction to the signal processing in life sciences, sampling, filtering, analogue-to-digital conversion. Application of the knowledge for medical practice: electrocardiography (ECG), electromyography (EMG).

Assumed knowledge: Basic level from physics, biology and informatics

Method: After theoretical introduction, measurements by using Biopac system are performed followed by computer-assisted evaluations.

Preliminary programme:

- Introduction to the signal acquisition and processing, filtering, AD conversion
- Biophysical background of electrophysiological processes
- Measurement and evaluation of ECG and EMG

Tools:

- Biopac measurement system, computers

Notes, handouts:

- Lecture and practical presentations

Other literature

2. Damjanovich S., Fidy J., Szöllősi J. (Eds.) Medical Physics, Medicina, 2009

Basic biostatistics with applications

Lecturer: Krisztina Boda PhD, Associate Professor

Audience: teacher-secondary school student group

Language: Hungarian - English

Planned length: 3 hours in a teaching cabinet, 10 hours self-learning

Goal: The subject is designed to give basic biostatistical knowledge often used in medical research and to learn modeling and interpreting results of computer programs. The main purpose is teaching students how to find the most appropriate method to describe and present their data and to find significant differences or associations in the data set. Although the course gives only an introduction to the basic biostatistical methods, it gives only an outline to the more complicated methods as well. By the end of the course students will be able to use the software in their own work and they will know some didactic aspects of the teaching.

Assumed knowledge: basic elementary mathematics

Method: Practical lessons will be held in computer cabinet. Statistical computer program will be used to compute statistical tests and to produce graphs. Simple problems must be solved also by hand calculations and the results can be checked by the computer program.

Topics:

- The aim of biostatistics, types of data, descriptive statistics.
- Problem solving: calculation of descriptive statistics from a few data
- Data bases in the SPSS program, preparing simple figures based on descriptive statistics
- Hypothesis tests: principle, steps, significance, meaning of the p-value
- Basic simple statistical tests (*t*-tests, khi-square tests, linear regression)
- Running the tests in SPSS, verifying assumptions, interpretation of results

Some topics for practice

- Step-by step description of the evaluation of given data
- Didactic aspects, interesting problems

Tools:

- *SPSS for Windows*

Notes, handouts:

- Electronic interactive note
- Test sheets
- Illustrations

Literature

1. Reiczigel Jenő, Harnos Andrea, Solymosi Norbert: Biostatisztika nem statisztikusoknak. PARS Kft. Nagykovácsi, 2007.

Building a conference center or triangulating surfaces

Lecturer: Géza Makay PhD, Associate Professor

Audience: High school students, teachers

Language: Hungarian

Planned length: 2 hours

Goal: To give some mathematical background of triangulating surfaces for buildings created using metal beam supports and glass triangles.

Assumed knowledge: Coordinate geometry, scalar product, vector product, cosine theorem, Euler's polyhedron theorem

Method: The topics of this lecture is the triangulation of surface of the conference center in Pécs built from metal support beams and glass triangles by the order of the Alukonstrukt Ltd, Szeged. This triangulation must be accomplished so that it agrees to the technical requirements of the building process, for example: the degree of the vertices inside the surface must be 5 or 6, there are fixed vertices and edges, and there are limits on the length of the support beams and the size of the angles of the triangles. There are no papers in

the literature concerning such a triangulation, so a completely new algorithm had to be invented.

Preliminary programme:

- Basic restrictions on the support beams and glass triangles
- Coordinate geometry, scalar product, vector product, cosine theorem
- Computing normal vectors, lengths, angles
- Refining the surface
- Restriction on the degree's of the vertices, Euler's polyhedron theorem
- Illumination by a spot lamp
- Projection for viewing the surface
- Creating the triangulation

Tools:

- computer

Demonstration experiments with a sound card

Lecturers: Zoltán Gingl, Professor of Physics
Péter Makra, Assistant Professor

Audience: secondary-school teachers and students

Language: English

Planned length: 2 hours

Goals: Most computers are equipped with analogue inputs that are ideal for the purposes of demonstration experiments: the inputs of the sound card. Though the sound card is not a measurement device, high sampling frequency and time-domain accuracy are natural requirements for a sound card to perform its function, and these properties serve certain measurements equally well. This way, the creative use of the sound card can substitute for educational aids that are often expensive and difficult to obtain. The aim of this course is to provide an overview of the potential of using the sound card in demonstration experiments, including the methods and pitfalls of sound card measurements.

Assumed knowledge: computer skills, basic electronics skills, Fourier analysis, alternating current circuits

Method: After outlining the technical background, I shall show examples from measuring the speed of sound through photogates used in kinematics to homemade plethysmography, all of which can be realised with a sound card and a few cheap components. Discussing the principles of sound card measurements also offers an opportunity to introduce essential communication methods such as amplitude modulation or frequency modulation.

Preliminary programme:

- How can we use the sound card for the purposes of measurements?
- pitfalls of AC coupling
- impedance measurement principle
- the principle of amplitude modulation
- the principle of frequency modulation
- sample experiments

Tools:

- computer with a sound card, beamer, self-made software, microphone and various home-made hardware

Notes, handouts:

- Lecture slides and freely downloadable measurement software.

Other literature

3. Zoltán Gingl, Róbert Mingesz, Péter Makra, János Mellár, ‘Review of sound card photogates’. arXiv:1103.1760v1
4. http://www.noise.physx.u-szeged.hu/EduDev/Photogate/

Computer-aided, exploration centered teaching of math in the practice II. Experimental Mathematics in Action in the Classroom

Lecturer: Attila Máder, teaching assistant, high school teacher

Audience: Mathematics teachers, students

Language: English/Hungarian

Planned length: 2 hours

Goal: To give an introduction to the filed Experimental Mathematics, and show how computer supported methods work in teaching via examples.

Assumed knowledge: -

Method: Examples of new teaching methods with the help of computer applications, which will be available for the participants for further study.

Preliminary programme:

- Discovering identities behind complete induction with OEIS: http://oeis.org/
- Computer as a tool: set theory identities, the Monty Hall-Problem, dicing
- Computer as a partner: discovering elementary function transformations, trigonometric identities, Thales' theorem, Taylor polynomials and series, definite integral of a function and the area of the region bounded by its graph
- Islands, questions and answers: how experimental mathematics works

Tools:

- beamer, computer
- Mathematica, Autograph, GeoGebra, Flash, WWW

Notes, handouts:

- Prepared interactive projects in LaTex
- Lecture presentations in LaTex

Other literature

1. George E. Andrews: The Death of proof? Semi-Rigorous Mathematics? You've Got to Be Kidding!, The mathematical intelligencer 16(4), 16-18.
2. C. N. Barton: Autograph Activities - Teacher Demonstrations for 16-19, Eastmond Publishing Ltd.
3. Jonathan Borwein, David Bailey: Mathematics by Experiment: Plausible Reasoning in the 21st Century, A K Peters, Natick, Massachusetts, 2004.
4. Steven G. Krantz: The Proof is in the Pudding: The Changing Nature of Mathematical Proof, Springer-Verlag, 2010
5. Attila Máder, Róbert Vajda: Elementary Approaches to the Teaching of the Combinatorial Problem of Rectangular Islands, International Journal of Computers for Mathematical Learning 15/3
6. György Pólya: Mathematical Discovery I-II, John Wiley & Sons, New York, 1965.

Free software in mathematics education

Lecturer: Zoltán Kovács Assistant Professor

Audience: teacher-grammar school student groups

Language: Hungarian - English

Planned number of lectures: **4 hours** in computer lab, 20 hours of homework

Aim of course: The students should learn freely available software packages in mathematics that may enhance the quality of teaching, mainly in visualization and calculation speedups. The final aim is to have the students the skill to work with mathematics computer software on their own.

Prerequisities: introductory analysis, algebra, geometry, probability theory, number theory; spreadsheets, basic knowledge of working with operating systems

Method: The courses take place in a computer lab. Solution of practical problems will follow the summary of theory. Finally, the students will solve exercises on their own with the help of the lecturer.

Outline:

- Computer algebra systems (wxMaxima, matek.hu, MathematicaPlayer, Sage).

- Dynamic geometry systems, GeoGebra.
- Spreadsheets, OpenOffice.org/LibreOffice.
- Word processing and mathematics (LyX).
- Figures (Asymptote).

Applied tools:

- *wxMaxima, web browsers, GeoGebra, OpenOffice.org/LibreOffice, LyX, Asymptote*

Bibliography:

- http://wmi.math.u-szeged.hu/mediawiki/index.php/Számítógép_alkalmazása_a_matematika_tanításában

Summary of the lectures and courses of the school in Novi Sad

Group of researchers and PhD. students

Introduction to Traffic Flow Theory

Lecturer: Arpad Takači, PhD, Professor of Mathematics

Audience: Mathematics PhD students, researchers

Language: English

Planned length: 2 hours

Goal: Teaching contribution: Several traffic flow mathematical models are visualized by using the *AnyLogic* Russian modeling package.

Assumed knowledge: ODEs, PDEs

Method: Examples of convolutions, fractional derivatives, delta sequences are considered with the help of the *GeoGebra* package.

Preliminary programme:

- introduction to traffic flow theory
- traffic after the green lights,
- traffic after the red lights,
- road congestion
- shock waves, rarefaction waves
- analysis of accidents and other models

Tools:

- video projector, computer
- *AnyLogic*

Notes, handouts:

- Prepared interactive projects in *AnyLogic*

Other literature

1. Dj.Takači, A. Takači, Partial Differential Equations through Examples and Exercises, Kluwer Academic Publishers, Dordrecht, 1997.
2. A. Takači, Mathematical Modeling (in Serbian), Faculty of Sciences, Novi Sad, and WUS, Belgrade, 2006.
3. Mathematical Modeling, Mark M. Meerschaert, Second Edition, Academic Press, 1999.

Some applications of impulsive systems

Lecturer: János Karsai PhD, Associate Professor

Audience: Mathematics PhD students, researchers

Language: English

Planned length: 2 hours

Goal: To give an introduction to the notion, basic properties and some methods of the study of impulsive systems via examples.

Assumed knowledge: Calculus, differential equations

Method: Examples of impulsive phenomena are considered theoretically and with the help of computer applications, which will be available for the participants for further study.

Preliminary programme:

- Systems with fixed and varying instants of impulses; state-dependent impulses
- Elementary concepts: direction field, impulse field, solutions
- Visualization of impulsive systems
- Some properties unexpected
- Applications: repeated drug dosing, swinging, switches, bouncing ball

Tools:

- beamer, computer
- Mathematica

Notes, handouts:

- Prepared interactive projects in Mathematica
- Lecture presentations in Mathematica

More literature

1. Karsai J., Study of impulsive phenomena, Mathematica experiments, Typotex 2002

Some experiences using multivariate modeling methods en paediatric aneasthesia

Lecturer: Krisztina Boda PhD, Associate Professor

Audience: Mathematics PhD students, researchers

Language: English

Planned length: 2 hours

Goal: To give an introduction to the multivariate modeling and using statistical software packages via examples.

Assumed knowledge: basic probability theory, basic biostatics

Method: summary of theory, application to medical data, outline to the use of SPSS or other statistical packages (SAS, Statistica for Windows, R).

Preliminary programme:

- Theory: types of studies, comparing two proportions, general linear models, generalized linear models, logistic regression, relative risk regression.
- Applications: modeling the risk factors of respiratory complications. Multiplicity problems.
- Practice: the use of SPSS (and others statistical software packages)

Tools:

- beamer, computer
- SPSS

Notes, handouts:

- Lecture presentations

Other literature

1. Agresti A. Categorical Data Analysis. Wiley, 2002.

2. von Ungern-Sternberg BS. - Boda K. - Schwab C. - Sims C. - Johnson C. - Habre W.: Laryngeal mask airway is associated with an increased incidence of adverse respiratory events in children with recent upper respiratory tract infections. Anesthesiology 107(5):714-9, 2007.
3. von Ungern-Sternberg BS. - Boda K. - Chambers NA. - Rebmann C. - Johnson C. - Sly PD. - Habre W. Risk assessment for respiratory complications in paediatric anaesthesia: a prospective cohort study. LANCET 376, 773-783, 2010.

On the visualizations of generalized functions I

Lecturer: Stevan Pilipović, PhD, Full Professor, Academician

Audience: Mathematics PhD students, researchers

Language: English

Planned length: 2 hours

Goal: Teaching contribution: Visualization and presentations of the main notions from the theory of generalized functions, which will enable their better understanding.

Assumed knowledge: Calculus, functional analysis

Method: Examples of convolutions, fractional derivatives, delta sequences are considered.

Preliminary programme:

- The properties of the generalized functions;
- Distribution;
- Delta sequences

Tools:

- beamer, computer

Notes, handouts:

- Prepared interactive projects .

Literature:

1. Carmichael, R., Kaminski, A., Pilipović, S., Boundary Values and Convolution in Ultradistribution Spaces, ISAAC Series on Analysis Applications and Computations -Vol. 1, World Scientific, 2007.
2. S. Pilipović, B. Stanković, Prostori Distribucija, Srpska Akademija Nauka i Umetnosti, Ogranak u Novom Sadu, Novi Sad, 2000.
3. S. Pilipović "Contrabutionsto local and microlocal analysis, an overwiew" Bull.Cl.Sci.Math. Nat. 35(2010) 79-95

Some physical examples

Lecturer: Marko Nedeljkov Professor of Mathematics

Planned length: 2 hours

Summary: There is no precise definition of wave, but one can describe it as a signal travelling from one place to another one with clearly visible speed. The signal can be any disturbance, like some kind of maxima or change of some quantity. Here, we will consider some simpler one-space-dimensional nonlinear problems and their simple wave solutions. We were dealing mainly with conservation laws which represents basic physical laws -- the most important building blocks in science. The prototype is so called the *continuity equation,* or the law of mass conservation.

On the probability and stochastic models

Lecturer: Danijela Rajter-Ćirić, PhD, Associate Professor

Audience: Mathematics PhD students, researchers

Language: English

Planned length: 2 hours

Goal: Teaching and presentations of the probability theory and stochastic analysis methods, main notions and examples

Assumed knowledge: Calculus, basic Probability

Method: Presentation of basic notions in probability theory through examples

Preliminary programme:

- Basics of probability theory
- Random variables
- Expectation and variance

Tools:

- slides, computer, blackboard

Literature:

1. D. Rajter-Ćirić, *Probability* University of Novi Sad, 2009. (In Serbian)
2. S. Ross, *Introduction to Probability Models,* Academic Press, 2003.
3. D. Rajter-Ćirić, *Stochastic Analysis*, notes for students.

On the visualizations of generalized functions II

Lecturer: Đurđica Takači, PhD, Professor of Mathematics

Audience: Mathematics PhD students, researchers

Language: English

Planned length: 2 hours

Goal: Teaching contribution: Visualization and presentations of the introduction to the main notions of the theory of generalized functions by using computer, which will enable their better understanding.

Assumed knowledge: Calculus, functional analysis

Method: Examples of convolutions, fractional derivatives, delta sequences are considered with the help of package *GeoGebra*.

Preliminary programme:

- The properties of the convolution
- operators
- fractional derivatives
- delta sequences

Tools:

- beamer, computer
- *GeoGebra*

Notes, handouts:

- Prepared interactive projects in GeoGebra

Literature:

1. Dj.Takači, A. Takači, N. Budinski, *"On Visualization Problems by using GeoGebra and Scientific Workplace"* The International Journal for Technology in Mathematical education, V17, No 4, 2011.
2. Dj.Takači, A. Takači, Partial Differential Equations through Examples and Exercises, Kluwer Academic Publishers, Dordrecht, 1997.
3. http://www.tech.plym.ac.uk/research/mathematics_education/field%20of%20work/ijtme/volume%2017/number_four.html

Fuzzy sets and logic with applications in natural sciences I

Lecturer: Branimir Šešelja, PhD, Professor of Mathematics

Audience: Mathematics and Natural Sciences PhD students, researchers

Language: English

Planned length: 2 hours

Goal: Teaching contribution: Presentation of main topics in lattices and lattice valued fuzzy structures.

Assumed knowledge: Basics in sets, relations and orderings

Method: Developing lattices from order, throughout main examples in sciences. Analysis of diagrams. Introduction to lattice valued structures and their properties.

Preliminary programme:

- Ordered sets and lattices: short introduction with characteristic examples
- Fuzzy sets: poset and lattice-valued ones
- Operations and relations on fuzzy power set
- Fuzzy relations: general, similarity and order
- Working with lattice valued structures: from real interval to residuated lattice

Tools:

- beamer, computer

Notes, handouts:

- Prepared in beamer; instructions for making diagrams of lattices.

Other: *Prepared together with prof. Andreja Tepavčević*

Literature:

1. B. Šešelja, A. Tepavčević, *Completion of Ordered Structures by Cuts of Fuzzy Sets, An Overview*, Fuzzy Sets and Systems 136 (2003) 1-19.
2. B. Šešelja, A. Tepavčević, *Representing Ordered Structures by Fuzzy Sets, An Overview*, Fuzzy Sets and Systems 136 (2003) 21-39.
3. B.A. Davey, H.A. Pristley, *Introduction to Lattices and Order*, Cambridge University Press, Cambridge, 1992.
4. G. Klir, B.Yuan, *Fuzzy sets and fuzzy logic*, Prentice-Hall PTR, New Jersey, 1995.

Fuzzy sets and logic with applications in natural sciences II

Lecturer: Andreja Tepavčević, PhD, Professor of Mathematics

Audience: Mathematics and Natural Sciences PhD students, researchers

Language: English

Planned length: 2 hours

Goal: Teaching contribution: Properties and applications of lattices and lattice valued fuzzy structures in various fields.

Assumed knowledge: Basics in sets, relations and orderings.

Method: Presentation of main topics from lattice valued structures and their applications.

Preliminary programme:

- Dealing with concepts (social sciences)
- Distribution of species (biology)
- preparing diagnosis (medicine)
- Fuzzy control (technology and informatics)
- Fuzzy cluster analysis (various fields)

Tools:

- beamer, computer

Notes, handouts:

- Prepared in beamer; Web examples.

Literature:

1. B. Šešelja, A. Tepavčević, *Completion of Ordered Structures by Cuts of Fuzzy Sets, An Overview*, Fuzzy Sets and Systems 136 (2003) 1-19.
2. B. Šešelja, A. Tepavčević, *Representing Ordered Structures by Fuzzy Sets, An Overview*, Fuzzy Sets and Systems 136 (2003) 21-39.
3. B.A. Davey, H.A. Pristley, *Introduction to Lattices and Order*, Cambridge University Press, Cambridge, 1992.
4. G. Klir, B.Yuan, *Fuzzy sets and fuzzy logic*, Prentice-Hall PTR, New Jersey, 1995.

Mathematical background of relational data model

Lecturer: Srđan Škrbić, PhD, Assistant Professor

Audience: PhD students, researchers

Language: English

Planned length: 2 hours

Goal: Teaching contribution: Introduction to relational model's theoretical background, together with implications these concepts make to practical usage.

Assumed knowledge: Relational data model, basic algebra

Method: Presentation of basic ideas, together with real-world examples.

Preliminary programme:

- Mathematical definitions of basic notions related to relational data model
- Armstrong's axioms
- Normal forms
- Normalization algorithms

Tools:

- beamer, computer

Notes, handouts:

- Prepared slides

Literature:

1. Edgar Frank Codd. A relational model of data for large shared data banks. Communications of the ACM, 13(6):377–387, 1970.
2. Christopher J. Date. An Introduction to Database Systems, Eighth Edition. Addison Wesley, July 2003.
3. Hector Garcia-Molina, Jeffrey D. Ullman, and JenniferWidom. Database Systems: The Complete Book. Prentice Hall Press, Upper Saddle River, NJ, USA, 2 edition, 2008.
4. A. Silberschatz, H. Korth, and S. Sudarshan. Database Systems Concepts. McGraw-Hill, Inc., New York, NY, USA, 5th edition, 2006.

Teachers, didactic PhD. students, high school students

Methods for solving Sudoku puzzles; Generating and solving Sudoku puzzles by computer Sudoku competition

Lecturer: Géza Makay PhD, associate professor

Audience: High school students, teachers

Language: English

Planned length: 2 hours

Goal: To give some mathematical background of the Sudoku puzzles, to present solution methods and an algorithm to generate such puzzles.

Assumed knowledge: Basic combinatorics, logic

Method: Some preliminary notes on Sudoku history, background, the mathematics of the Sudoku puzzles, examples of the solution methods. Solving Sudoku puzzles by computer (the brute force method and implementing solving methods), generating Sudoku puzzles.

Preliminary programme:

- The history/background of Sudoku
- The mathematics of Sudoku

- Methods for solving Sudoku puzzles
- Generating and solving Sudoku puzzles by computer
- Sudoku competition

Tools:
- computer
- paper, pen/pencil

Notes, handouts:
- Some Sudoku problems of different difficulty
- A description of some Sudoku solving methods

Other literature
- http://www.math.u-szeged.hu/Sudoku/

Computer-aided, exploration centered teaching of math in the practice II.
Experimental Mathematics in Action in the Classroom

Lecturer: Attila Máder, teaching assistant, high school teacher

Audience: Mathematics teachers, students

Language: English/Hungarian

Planned length: 2 hours

Goal: To give an introduction to the filed Experimental Mathematics, and show how computer supported methods work in teaching via examples.

Assumed knowledge: -

Method: Examples of new teaching methods with the help of computer applications, which will be available for the participants for further study.

Preliminary programme:
- Discovering identities behind complete induction with OEIS: http://oeis.org/
- Computer as a tool: set theory identities, the Monty Hall-Problem, dicing
- Computer as a partner: discovering elementary function transformations, trigonometric identities, Thales' theorem, Taylor polynomials and series, definite integral of a function and the area of the region bounded by its graph
- Islands, questions and answers: how experimental mathematics works

Tools:
- beamer, computer
- Mathematica, Autograph, GeoGebra, Flash, WWW

Notes, handouts:
- Prepared interactive projects in LaTex
- Lecture presentations in LaTex

Other literature

7. George E. Andrews: The Death of proof? Semi-Rigorous Mathematics? You've Got to Be Kidding!, The mathematical intelligencer 16(4), 16-18.
8. C. N. Barton: Autograph Activities - Teacher Demonstrations for 16-19, Eastmond Publishing Ltd.
9. Jonathan Borwein, David Bailey: Mathematics by Experiment: Plausible Reasoning in the 21st Century, A K Peters, Natick, Massachusetts, 2004.
10. Steven G. Krantz: The Proof is in the Pudding: The Changing Nature of Mathematical Proof, Springer-Verlag, 2010
11. Attila Máder, Róbert Vajda: Elementary Approaches to the Teaching of the Combinatorial Problem of Rectangular Islands, International Journal of Computers for Mathematical Learning 15/3
12. György Pólya: Mathematical Discovery I-II, John Wiley & Sons, New York, 1965.

On the definition of the logarithmic function, mathematical modelling of earthquake

Lecturer: Đurđica Takači, Professor

Audience: High school teachers, high school students

Language: Serbian

Planned length: 2 hours

Abstract: Graphs of function by using GeoGebra, defined by integrals, in particular logarithmic functions; logarithmic functions as mathematical model for l **earthquake**.

Introduction to fractional calculus with the applications

Predavač: Teodor Atancković, PhDAcademician

Audience: High school teachers, high school students

Language: Serbian

Planned length: 2 hours

Abstract: Introduction to fractional derivative and its application connection for high school contents.

Matematički modeli u nastavi matematike i fizike
Mathematical models from physics, with the accent on the high school contents

Lecturer: Marko Nedeljkov, PhD, Professor

Audience: High school teachers, high school students

Language: Serbian

Planned length: 2 hours

Abstract: Matematical models of the problems from physics appearing in high school, with the special accent on connection mathematics and physics contents in high school.

On the visualization of recurrent defined sequences by using GeoGebra

Lecturer: Duška Pešić, PhD, Professor of high school

Audience: High school teachers, high school students

Language: Serbian

Planned length: 2 hours

Abstract: Modern didactical approach to the examining functions by using computer, in particular GeoGebra, recurrent defined sequences, cobweb diagrams.

Rasplinuti skupovi u nastavi I
A generalization of characteristic, fuzzy sets and fuzzy relations, their basics properties I

Lecturer: Šešelja Branimir PhD, Professor

Audience: High school teachers, high school students

Language: Serbian

Planned length: 2 hours

Abstract: A generalization of characteristic, fuzzy sets and fuzzy relations, their basics properties I

Fuzzy sets in teaching II

Applications of fuzzy sets and fuzzy relations in contemporary technology (fuzzy controllers in washing machines, videocameras, etc.)

Lecturer: Andreja Tepavčević, PhD, Professor

Audience: High school teachers, high school students

Language: Serbian

Planned length: 2 hours

***Abstract*:** Applications of fuzzy sets and fuzzy relations in contemporary technology (fuzzy controllers in washing machines, camcorder, etc.)

Mathematical modeling and simulation in teaching mathematics

Lecturer: Arpad Takači, PhD, Professor

Audience: High school teachers, high school students

Language: Serbian

Planned length: 2 hours

Abstract: Mathematical models: Lorenz model of weather, epidemics model. Population dynamic model (predator-prey competing population), mathematical pendulum, bouncing of the ball.

Data basis approach in high school

Lecturer: Srđan Škrbić, PhD, Asistant professor

Audience: High school teachers, high school students

Language: Serbian

Planned length: 2 hours

Abstract: Data base approach, based on the corresponding algebra, adapted for the contents of the high school curriculum.

Didactical approach of the teaching statistic in high school

Lecturer: Danijela Rajter Ćirić, PhD, Associate professor

Audience: High school teachers, high school students

Language: Serbian

Planned length: 2 hours

Abstract: Different didactical approach of the teaching statistic in high school, concerning curriculum.